MATHEMATICS-X

MODULE-6

INDEX

CIRCLES

★ **INTRODUCTION**

In class IX, we have studied that a circle is a collection of all points in a plane which are at a constant distance from a fixed point. The fixed point is called the centre and the constant distance is known as the radius. We have also studied various terms related to a circle like chord, segment, sector, arc etc. Now, we shall study properties of a line touching a circle at one point.

★ **RECALL**

Circle

A circle is the locus of a point which moves in such a way that

it is always at the constant

distance from a fixed point in the plane.

The fixed point 'O' is called the centre of the circle. **The constant distance** 'OA' between the centre (O) and the moving point (A) is called the **Radius** of the circle.

Circumference

The distance round the circle is called the circumference of the circle.

$2\pi r$ = circumference of the circle

= Perimeter of the circle.

= boundary of the circle

r is the radius of the circle.

Chord

The chord of a circle is a line segment joining any two points on

the circumference. AB is the chord of the circle with centre O. In fig. AB is the chord of the circle.

Diameter

A line segment passing through the centre of the circle and having

its end points on the circle is called diameter. If r is the radius of the

circle then the diameter of the circle is twice the radius i.e., d = 2r

AOB is a diameter of the circle whose centre is O

AOB = OA + OB = r + r = 2r.

Arc of a circle

If P and Q be any two points on the circle then the circle is divided

into two pieces each of which is an arc. Now we denote the arc from

P to Q in counter clock-wise direction by PQ and the arc from Q to P
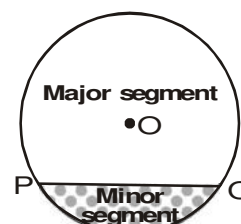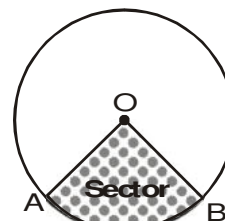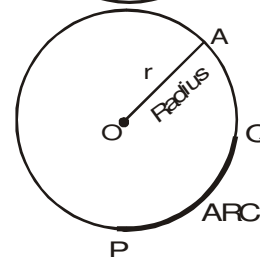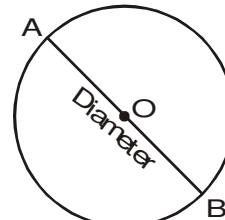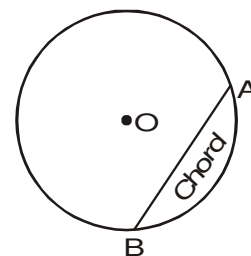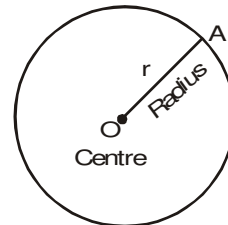
in clock-wise direction by QP.

Sector of a circle

The part of a circle bounded by two radii and arc is called sector.

In fig, the part of the plane region enclosed by AB and its

bounding radii OA and OB is a sector of the circle with centre O.

Segment of a circle

Let PQ be a chord of a circle with centre O and radius r, then PQ

divides the region enclosed by the circle into two parts. Each

part is called a segment of the circle. The part containing the

minor arc is called the **minor segment** and the part containing

the major arc is called the **major segment**.

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★ **INTERSECTION OF A CIRCLE AND A LINE**
Consider a circle with centre O and radius r and a line PQ in a plane. We find that there are three different positions a line can take with respect to the circle as given below in fig.
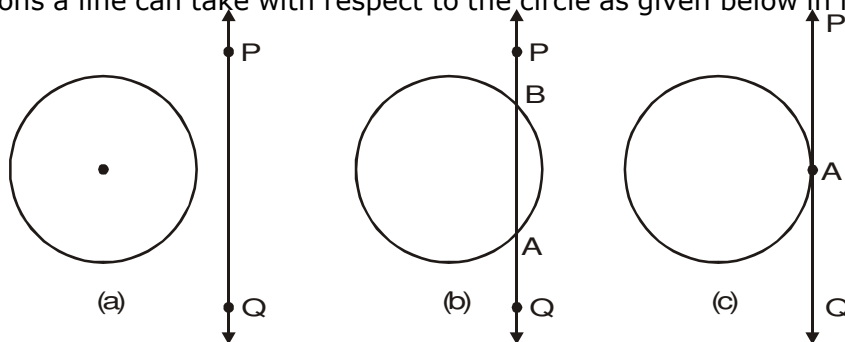
(a) The line PQ does not intersect the circle.
    In fig. (a) the line PQ and the circle have no common point. In this case PQ is called a non-intersecting line with respect to the circle.
(b) The line PQ intersect the circle in more than one point. In fig. (b), there are two common points A and B between the line PQ and the circle and we call the line PQ as a secant of the circle.
(c) The line intersect the circle in a single point i.e. the line intersect the circle in only one point. In fig. (c) you can verify that there is only one point 'A' which is common to the line PQ in the given circle. In this case the line is called a tangent to the circle.
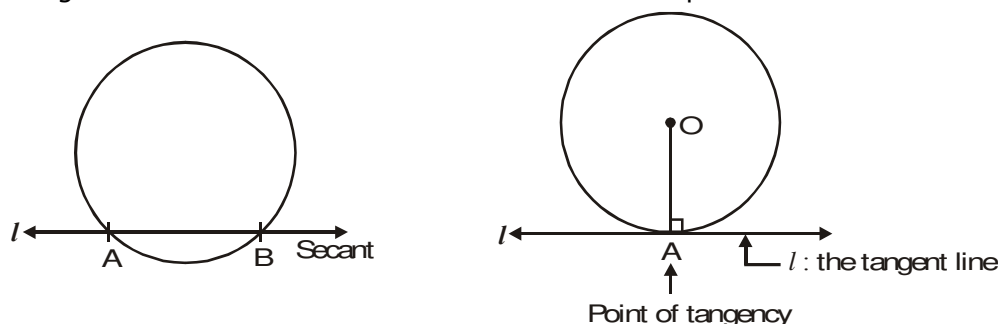
Tangent
A tangent is a straight line that meets the circle at one and only one point. This point 'A' is called point of contact or point of tangency in fig. (c).

Tangent as a limiting case of a secant
In the fig. the secant ℓ cuts the circle at A and B. If this secant ℓ is turned around the point A, keeping A fixed then B moves on the circumference closer to A. In the limiting position, B coincides with A. The secant ℓ becomes the tangent at A. Tangent to a circle is a secant when the two end points of its corresponding chord coincide.
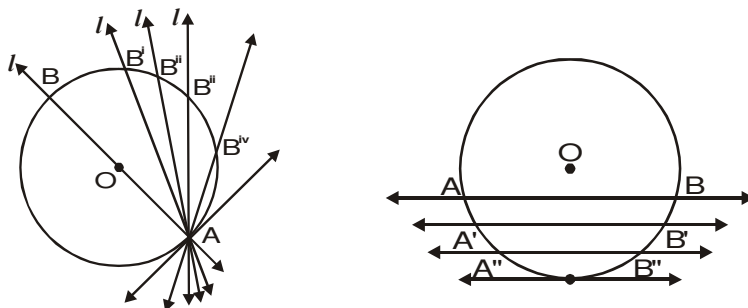
In the fig. ℓ is a secant which cuts the circle at A and B. If the secant is moved parallel to itself away from the centre, then the points A and B come closer and closer to each other. In the limiting position, they coincide into a single point at A, the secant ℓ becomes the tangent at A. Thus a tangent line is the limiting case of a secant when the two points of intersection of the secant and a circle coincide with the point A. The point A is called the point of contact of the tangent. The line ℓ touches the circle at the point A. i.e., the common point of the tangent and the circle is called the point of contact and the tangent is said to touch the circle at the common point.
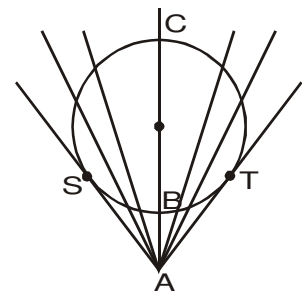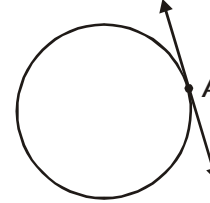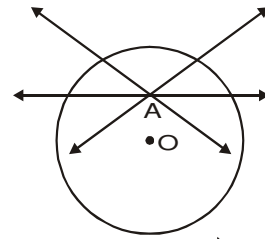

Note : The line containing the radius through the point of contact is called normal to the circle at the point.

★ **NUMBER OF TANGENTS TO A CIRCLE FROM A POINT**

1. If a point A lies inside a circle, no line passing through 'A' can be a tangent to the circle. i.e., No tangent can be drawn from the point A.

2. If A lies on the circle, then one and only one tangent can be drawn to pass through 'A'. i.e. Exactly one tangent can be drawn through A.

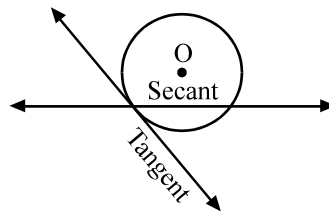**3.** If A lies outside the circle then exactly two tangents can be drawn through 'A'.
In the fig., a secant ABC is drawn from a point 'A' outside the circle, if the secant is turned around A in the clockwise direction, in the limiting position, it becomes a tangent at T. Similarly if the secant is turned in the anti-clockwise direction, in the limiting position, it becomes a tangent at S. Thus from a point A outside a circle only two tangents can be drawn. The points S and T where the lines touch the circle are called the points of contact.

I. Secant :
A line which intersects a circle in two distinct points is called a secant.

II. Tangent :
A line meeting a circle only in one point is called a tangent to the circle at that point. The point at which the tangent line meets the circle is called the point of contact.

Number of Tangents to a Circle
(i) There is no tangent passing through a point lying inside the circle.
(ii) There is one and only one tangent passing through a point lying on a circle.
(iii) There are exactly two tangents through a point lying outside a circle.

Length of Tangent
The length of the line segment of the tangent between a given point and the given point of contact with the circle is called the length of the tangent from the point to the circle.

Results on Tangents

Theorem 1 :
The tangent at any point of a circle is perpendicular to the radius through the point of contact.
Given : A circle with centre O and a tangent AB at a point P of the circle.
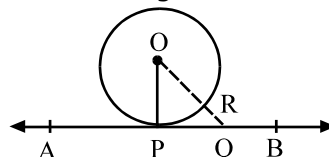
To prove : OP ⊥ AB.
Construction : Take a point Q, other than P, on AB. Join OQ.
Proof : Q is a point on the tangent AB, other than the point of contact P.
∴ Q lies outside the circle.
Let OQ intersect the circle at R.
Then, OR < OQ [a part is less than the whole]   .... (i)
But, OP = OR [radii of the same circle].   .... (ii)
∴ OP < OQ [from (i) and (ii)].

Thus, OP is shorter than, any other line segment joining O to any point of AB, other than P.

In other words, OP is the shortest distance between the point O and the line AB.

But, the shortest distance between a point and a line is the perpendicular distance.

∴ OP ⊥ AB.

Theorem 2 : (Converse of Theorem 1)

A line drawn through the end of a radius and perpendicular to it is a tangent to the circle.

Given : A circle with centre O in which OP is a radius and AB is a line through P such that OP ⊥ AB.
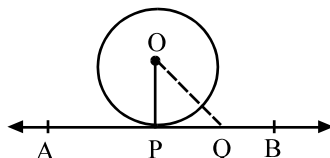
To prove : AB is a tangent to the circle at the point P.

Construction : Take a point Q, different from P, on AB. Join OQ.

Proof : We know that the perpendicular distance from a point to a line is the shortest distance between them.

∴ OP ⊥ AB ⇒ OP is the shortest distance from O to AB.

∴ OP < OQ.

∴ Q lies outside the circle

[∵ OP is the radius and OP < OQ].

Thus, every point on AB, other than P, lies outside the circle.
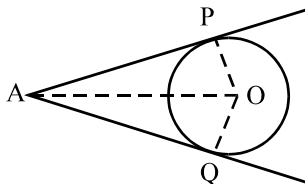
∴ AB meets the circle at the point P only.

Hence, AB is the tangent to the circle at the point P.

Theorem 3 :

The lengths of tangents drawn from an external point to a circle are equal.

Given : Two tangents AP and AQ are drawn from a point A to a circle with centre O.

To prove : AP = AQ

Construction : Join OP, OQ and OA.

Proof : AP is a tangent at P and OP is the radius through P.

∴ OP ⊥ AP.

Similarly, OQ ⊥ AQ.

In the right triangle OPA and OQA, we have

OP = OQ [radii of the same circle]

OA = OA [common]

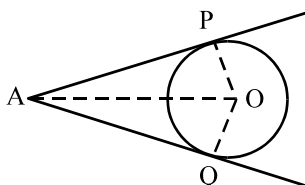∴ ΔOPA ≅ ΔOQA [by RHS–congruence]

Hence, AP = AQ.

Theorem 4 :

If two tangents are drawn from an external point then

(i) They subtend equal angles at the centre, and

(ii) They are equally inclined to the line segment joining the centre to that point.

Given : A circle with centre O and a point A outside it. Also, AP and AQ are the two tangents to the circle.

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To prove : ∠AOP = ∠AOQ and ∠OAP = ∠OAQ.
Proof : In ΔAOP and ΔAOQ, we have
AP = AQ [tangents from an external point are equal]
OP = OQ [radii of the same circle]
OA = OA [common]
∴ ΔAOP ≅ ΔAOQ [by SSS–congruence].
Hence, ∠AOP = ∠AOQ and ∠OAP = ∠OAQ.

SUMMARY OF THE CHAPTER

BASIC CONCEPTS AND IMPORTANT RESUTLS
* **Circle**
  A circle is a closed plane figure consisting of all those points of the plane which are at a constant distance from a fixed point in that plane. The fixed point if called its centre and the constant distance is called its radius.

* **Intersection of a line and a circle :** The following three cases arise :
  Case I. A line may not intersect a circle.
  Case II. A line may intersect a circle in two points.
  Case III. A line may intersect a circle in one point.
 Thus, a line can intersect a circle atmost in two points.
* A line Which intersects a circle in two points is called a secant.

* A line which intersects a circle in one point is called a tangent to the circle. Thus, a line which intersects a circle in one point is called a tangent to the circle. Thus, a line which intersects a circle in one and only one point is a tangent to the circle. The common point is called point of contact.
  In the adjoining figure, the line ℓ intersects the circle in only one point P. The line l is a tangent to the circle at P and P is the point of contact.
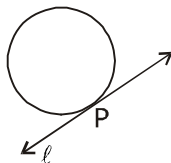
* Tangent to a circle is a special case of a secant,when the two ends of its corresponding chord coincide.
▸ The tangent at any point of a circle is perpendicular to the radius through the point of contact.
* If P is a point on a circle with centre C, then the line drawn through P and perpendicular to CP is the tangent to the circle at P.
* One and only one tangent can be drawn to a circle at a given point P on the circle with centre C because only one perpendicular can be drawn to CP through P.

* The tangents drawn at the ends of a diameter of a circle are parallel.

* The line segment joining the points of contact of two parallel tangens passes through the centre of the circle.

* Number of tangents to a circle from a point :
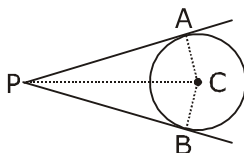 The following three cases arise :
  Case I. There is no tangent to a circle passing through a point lying inside the circle.
  Case II. There is one and only one tangent to a circle passing through a point lying on the circle.
  Case III. There are exactly two tangents to a circle passing through a point lying outside the circle.
 In the adjoining figure, P is an external point to a circle.
  PA and PB are two tangents to the circle, A and B are points of contact. The length of the segment PA (or PB) is called the length of the tangent.

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SOLVED PROBLEMS

Ex.1 A point A is 26 cm away from the centre of a circle and the length of tangent drawn from A to the circle is 24 cm. Find the radius of the circle.

Sol. Let O be the centre of the circle and let A be a point outside the circle such that OA = 26 cm.

Let AT be the tangent to the circle.

Then, AT = 24 cm. Join OT.
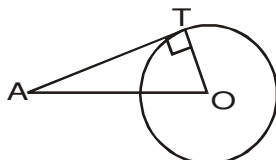
Since the radius through the point of contact is perpendicular to the tangent, we have ∠OTA = 90°. In right ∆OTA, we have $OT^2 = OA^2 - AT^2$

$= [(26)^2 - (24)^2] = (26 + 24)(26 - 24) = 100$.

$\Rightarrow$ OT = $\sqrt{100}$ = 10 cm.

Hence, the radius of the circle is 10 cm.

Ex.2 In the given figure, ∆ABC is right-angled at B, in which AB = 15 cm and BC = 8 cm. A circle with centre O has been inscribed in ∆ABC. Calculate the value of x, the radius of the inscribed circle.

Sol. Let the inscribed circle touch the sides AB, BC and CA at P, Q and R respectively. Applying Pythagoras theorem on right ∆ABC, we have

$AC^2 = AB^2 + BC^2 = (15)^2 + (8)^2 = (225 + 64) = 289$

$\Rightarrow$ AC = $\sqrt{289}$ = 17 cm.

Clearly, OPBQ is a square.

[∵ ∠OPB=90°, ∠PBQ=90°,

∠OQB=90° and OP=OQ= x cm]

∴ BP = BQ = x cm.

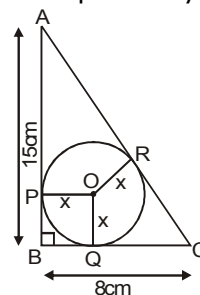Since the tangents to a circle from an exterior point are equal in length, we have AR = AP and CR = CQ.

Now,   AR = AP = (AB – BP) = (15 – x) cm

              CR = CQ = (BC – BQ) = (8 – x) cm.

∴   AC = AR + CR

$\Rightarrow$ 17 = (15 – x) + (8 – x) $\Rightarrow$ 2x = 6 $\Rightarrow$ x = 3.

Hence, the radius of the inscribed circle is 3 cm.

Ex.3 If all the sides of a parallelogram touch a circle, show that the parallelogram is a rhombus.

Sol. Let ABCD be a parallelogram whose sides AB, BC, CD and DA touch a circle at the points P, Q, R and S respectively.   **[NCERT]**

Since the lengths of tangents drawn from an external point to a circle are equal, we have

AP = AS, BP = BQ, CR = CQ and DR = DS.

∴ AB+CD = AP + BP + CR + DR

   = AS + BQ + CQ + DS

   = (AS+DS)+(BQ+CQ)

   = AD + BC

Now, AB + CD = AD + BC

$\Rightarrow$ 2AB = 2BC

   [∵ Opposite sides of a ⏥ gm are equal]

$\Rightarrow$ AB = BC

∴ AB = BC = CD = AD.

Hence, ABCD is a rhombus.

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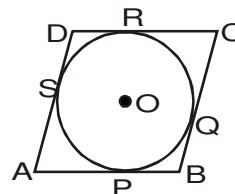
Ex.4 In the given figure, the in circle of ∆ABC touches the sides AB, BC and CA at the points P, Q, R respectively. Show that AP + BQ + CR
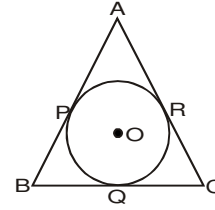
= BP + CQ + AR = $\frac{1}{2}$ (Perimeter of ∆ABC)

Sol. Since the lengths of two tangents drawn from an external point to a circle are equal, we have

AP = AR, BQ = BP and CR = CQ

∴ AP + BQ + CR = AR + BP + CQ   ...(i)

Perimeter of ∆ABC = AB + BC + CA

= AP + BP + BQ + CQ + AR + CR

= (AP + BQ + CR) + (BP + CQ + AR)

= 2(AP + BQ + CR) [Using (i)]

∴ AP + BQ + CR = BP + CQ + AR

= $\frac{1}{2}$ (Perimeter of ∆ABC).

Ex.5 In two concentric circles, prove that a chord of larger circle which is tangent to smaller circle is bisected at the point of contact.   **[NCERT]**

Sol. Let there be two concentric circles, each with centre O.
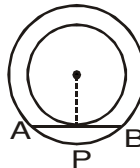
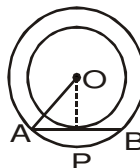Let AB be a chord of larger circle touching the smaller circle at P. Join OP.

Since OP is a radius of smaller circle and APB is a tangent to it at the point P, so OP ⊥ AB.

But the perpendicular from the centre to a chord, bisects the chord.

∴   AP = PB

Hence, AB is bisected at the point P.

Ex.6 Two concentric circles are of radii 13 cm and 5 cm. Find the length of the chord of the outer circle which touches the inner circle.

Sol. Let O be the centre of the concentric circles and let AB be a chord of the outer circle, touching the inner circle at P. Join OA and OP.

Now, the radius through the point of contact is perpendicular to the tangent.

∴   OP ⊥ AB.

Since, the perpendicular from the centre to a chord, bisects the chord, AP = PB. Now, in right ∆OPA, we have OA = 13 cm and OP = 5 cm.

∴ OP² + AP² = OA² ⇒ AP² = OA² − OP²

= (13² − 5²) = (169 − 25) = 144.

⇒ AP = $\sqrt{144}$ = 12 cm.

∴ AB = 2AP = (2 × 12) cm = 24 cm.

Hence, the length of chord AB = 24 cm.

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Ex.7 In the given figure, PT is a common tangent to the circles touching externally at P and AB is another common tangent touching the circles at A and B. Prove that:
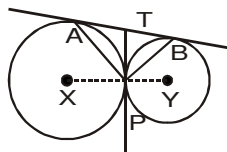
(i) T is the mid-point of AB

(ii) ∠APB = 90°

(iii) If X and Y are centres of the two circles,

show that the circle on AB as diameter touches the line XY.

Sol. (i) Since the two tangents to a circle from an external point are equal, we have

TA = TP and TB = TP.

∴ TA = TB [Each equal to TP]

Hence, T bisects AB, i.e., T is the mid-point of AB.

(ii) TA = TP $\Rightarrow$ ∠TAP = ∠TPA

TB = TP $\Rightarrow$ ∠TBP = ∠TPB

∴ ∠TAP + ∠TBP = ∠TPA + ∠TPB = ∠APB

$\Rightarrow$ ∠TAP + ∠TBP + ∠APB = 2∠APB

$\Rightarrow$ 2∠APB = 180°

[∵ The sum of the ∠s of a △ is 180°]

$\Rightarrow$ ∠APB = 90°

(iii) Thus, P lies on the semi-circle with AB as diameter.

Hence, the circle on AB as diameter touches the line XY.

Ex.8 Two circles of radii 25cm and 9cm touch each other externally. Find the length of the direct common tangent.

Sol. Let the two circles with centres A and B and radii 25 cm and 9 cm respectively touch each other externally at a point C.

Then, AB = AC + CB = (25 + 9) cm = 34 cm.

Let PQ be a direct common tangent to the two circles.

Join AP and BQ.

Then, AP ⊥ PQ and BQ ⊥ PQ.

[∵ Radius through point of contact is perpendicular to the tangent]

Draw, BL ⊥ AP. Then, PLBQ is a rectangle.

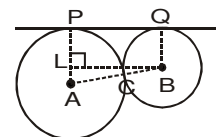Now, LP = BQ = 9 cm and PQ = BL.

∴ AL = (AP – LP) = (25 – 9) cm = 16 cm.

From right △ALB, we have

$AB^2 = AL^2 + BL^2 \Rightarrow BL^2 = AB^2 – AL^2 = (34)^2 – (16)^2 = (34 + 16)(34 – 16) = 900$

$\Rightarrow BL = \sqrt{900} = 30$ cm.
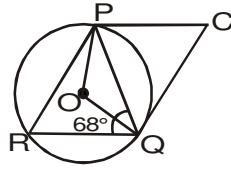
∴ PQ = BL = 30 cm.

Hence, the length of direct common tangent is 30 cm.

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Ex.9 In the given figure, PQ = QR, ∠RQP = 68°, PC and CQ are tangents to the circle with centre O. Calculate the values of : (i) ∠QOP (ii) ∠QCP

Sol. (i) In ΔPQR,

PQ = QR ⇒ ∠PRQ = ∠QPR

[∠s opp. to equal sides of a Δ are equal]

Also, ∠QPR + ∠RQP + ∠PRQ = 180°

[Sum of the ∠s of a Δ is 180°]

⇒   68° + 2∠PRQ = 180°

⇒   2∠PRQ = (180° – 68°) = 112°

⇒   ∠PRQ = 56°.

∴   ∠QOP = 2∠PRQ = (2 × 56°) = 112°. [Angle at the centre is double the angle on the circle]

(ii) Since the radius through the point of contact is perpendicular to the tangent, we have

∠OQC = 90° and ∠OPC = 90°.

Now, ∠OQC+∠QOP+∠OPC+∠QCP=360°

[Sum of the ∠s of a quad. is 360°)

⇒ 90° + 112° + 90° + ∠QCP = 360°.

⇒ ∠QCP = (360° – 292°) = 68°.

Ex.10 With the vertices of ΔABC as centres, three circles are described, each touching the other two externally. If the sides of the triangle are 9 cm, 7 cm and 6 cm, find the radii of the circles.

Sol. Let AB = 9 cm, BC = 7 cm and CA = 6 cm.

Let x, y, z be the radii of circles with centres A, B, C respectively.

Then, x + y = 9, y + z = 7 and z + x = 6.
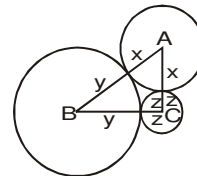
Adding, we get 2(x + y + z) = 22

⇒ x + y + z = 11.

∴ x = [(x + y + z) – (y + z)]

= (11 – 7) cm = 4 cm.

Similarly, y = (11 – 6) cm = 5 cm

and z = (11 – 9) cm = 2 cm.

Hence, the radii of circles with centres A, B, C are 4 cm, 5cm and 2 cm respectively.

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EXERCISE – I | UNSOLVED PROBLEMS

Q.1 Two tangents PA and PB are drawn to the circle with centre O, such that $\angle APB = 120^\circ$. Prove that OP = 2AP.

Q.2 Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre.

Q.3 In the figure, AB is diameter of a circle with centre O and QC is a tangent to the circle at C. If $\angle CAB = 30^\circ$, find (i) $\angle CQA$, (ii) $\angle CBA$.
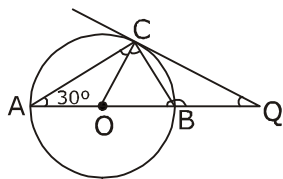
Q.4 From a point P, 10 cm away from the centre of a circle, a tangent PT of length 8 cm is drawn. Find the radius of the circle.

Q.5 In the given figure, PQ is a chord of length 8cm of a circle of radius 5cm. The tangents at P and Q intersect at a point T. Find the length TP.
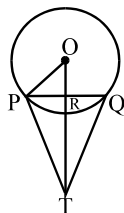
Q.6 Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.

Q.7 Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Q.8 Prove that the line segment joining the point of contact of two parallel tangents to a circle is a diameter of the circle.

Q.9 In the given figure, the incircle of $\triangle ABC$ touches the sides BC, CA and AB at D, E, F respectively.
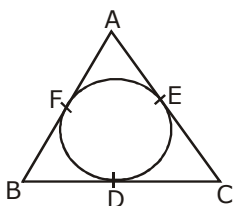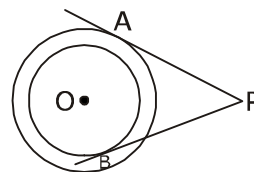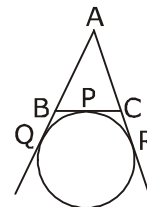

Prove that

AF + BD + CE = AE + CD + BF

$= \dfrac{1}{2}$ (perimeter of $\triangle ABC$)

Q.10 Prove that the lengths of the tangents drawn from an external point to a circle are equal.

Q.11 Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segments joining the points of contact at the centre.

Q.12 In the adjoining figure, two concentric circles with centre O are of radii 5 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, then find BP.
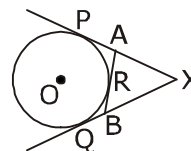

Q.13 (a) A circle is touching the side BC of a $\triangle ABC$ at P and is touching AB and AC when produced
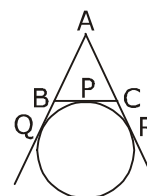
at Q and R respectively.

Prove that AQ = $\dfrac{1}{2}$ (perimeter of $\triangle ABC$).

(b) In the adjoining figure, XP and XQ are two tangents to a circle with centre O from a point X outside the circle. ARB is a tangent to the circle at R. Prove that XA + AR = XB + BR.
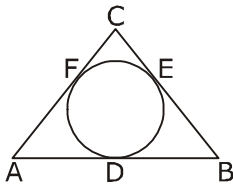

Q.14 In the adjoining figure, a circle touches the side BC of $\triangle ABC$ at P and touches AB and AC produced at Q and R respectively. If AQ = 5 cm, find the perimeter of $\triangle ABC$.


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Q.15 In the adjoiing figure, the incircle of △ABC touches the sides BC, CA and AB at the point P, Q, and R respectively. If AB = AC, prove that BP = CP.
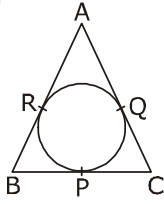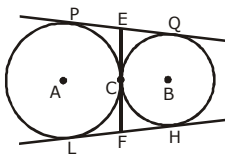
Q.16 In the adjoining figure, a circle is inscribed in a triangle ABC having sides BC = 8 cm, AC = 10 cm and AB = 12 cm. Find AD, BE and CF.

**Q.17** In the adjoining figure, quadrilateral ABCD is circumscribed. Find the value of x.
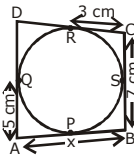
Q.18 In the adjoining figure, two circles touch each other externally at C. Prove that the common tangent at C bisects the other two common tangents.

Q.19 Prove that the line segment joining the points of contact of two parallel tangents passes through the centre.

Q.20 Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.

**Q.21** In the adjoining figure, AB is a diameter of the circle with centre O, AP, BQ and PQ are tangents to the circle. Prove that ∠POQ = 90º
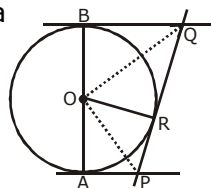
Q.22 Prove that the tangents drawn from an external point to a circle subtend equal angles at the centre of the circle.

Q.23 Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

Q.24 Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.
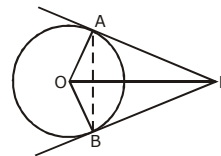
Q.25 In the adjoining figure, PA and PB are tangents drawn from an external point P to a circle with centre O. Prove that ∠APB = 2∠OAB.
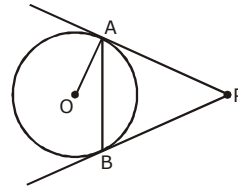
Q.26 Prove that a parallelogram circumscribing a circle is a rhombus.

Q.27 Prove that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Q.28 In the adjoining figure, PA and PB are tangents to a circle with centre O. If OP is equal to the diameter of the circle, prove that ABP is an equilateral triangle.

**Q.29** In the adjoining figure, a circle is inscribed in a quadrilateral ABCD in which ∠B =90º. If AD=23 cm, AB = 29 cm and DS = 5 cm. Find the radius (r) of the circle.
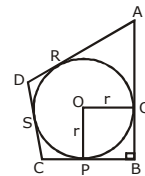
**Q.30** In the given figure PQ is a diameter of a circle with centre O and PT is a tangent at P. QT meets the circle at R. If ∠POR = 72°, find ∠PTR.
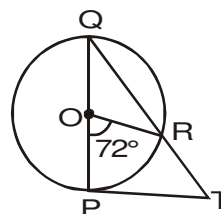
Q.31 In the given figure, PA and PB are the tangent segments to a circle with centre O. Show that the points A, O, B and P are concyclic.
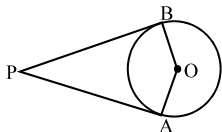

Q.32 From an external point P, tangents PA and PB are drawn to a circle with centre O. If CD is the tangent to the circle at a point E and PA = 14cm, find the perimeter of △ PCD.
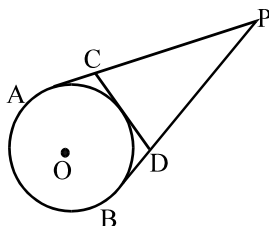

Q.33 A circle is inscribed in a △ABC having AB = 10 cm, BC = 12 cm and CA = 8 cm and touching these sides at D, E, F respectively, as shown in the figure. Find AD, BE and CF.
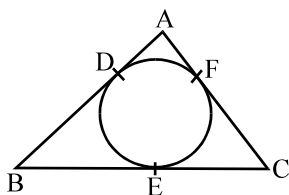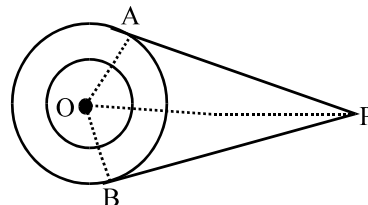

Q.34 In the given figure, ABCD is a quadrilateral in which ∠D=90°. A circle C(O, r) touches the sides AB, BC, CD and DA at P, Q, R , S respectively. If BC = 38 cm, CD = 25 cm and BP = 27 cm, find the value of r.
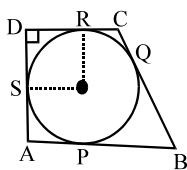

Q.35 Find the length of tangent drawn to a circle with radius 7 cm from a point 25 cm away from the centre of the circle.

Q.36 A point P is 26 cm away from the centre of a circle and the length of the tangent drawn from P to the circle is 24 cm. Find the radius of the circle.

Q.37 Two tangent segments BC and BD are drawn to a circle with centre O such that ∠CBD=120° . Prove that OB=2BC.
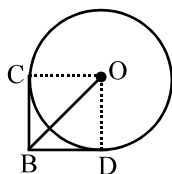
Q.38 In the given figure, O is the centre of two concentric circles of radii 4 cm and 6 cm respectively. PA and PB are tangents to the outer and inner circle respectively. If PA = 10 cm, find the length of PB upto one place of decimal.

Q.39 In the given figure, a circle touches all the four sides of a quadrilateral ABCD whose three sides are AB = 6 cm, BC = 7 cm and CD = 4cm. Find AD.
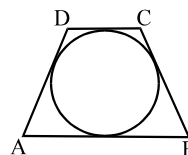
Q.40 In the given figure, PA and PB are tangents such that PA = 9 cm and ∠APB = 60°. Find the length of chord AB.
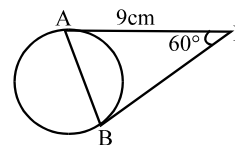
EXERCISE – II BOARD PROBLEMS

Q.1 In the given figure, TAS is a tangent to the circle, with centre O, at the point A. If ∠OBA = 32°, find the value of x. **[Delhi-1996C]**

Q.2 In the given figure, ABC is a right angled triangle right angled at A, with AB = 6cm and AC = 8 cm. A circle with centre O has been inscribed inside the triangle. Calculate the value of r, the radius of the inscribed circle. **[AI-1998]**
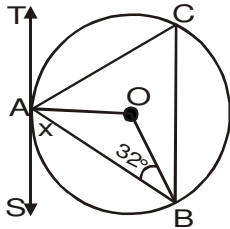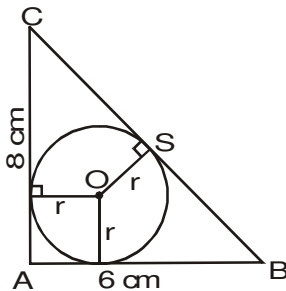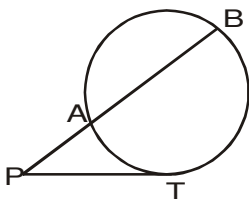
Q.3 In the given figure, PT is tangent to the circle at T. If PA = 4 cm and AB = 5 cm, find PT. **[Delhi-1998C]**

Q.4 In the figure, O is the centre of the circle, PQ is tangent to the circle at A. If ∠PAB = 58°, find ∠ABQ and ∠AQB. **[AI-1999C]**
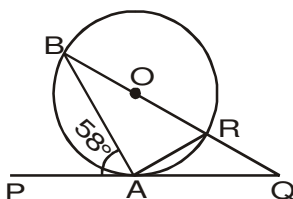
Q.5 In figure, a circle touches the side BC of ΔABC at P and touches AB and AC produced at Q and R respectively. If AQ = 5 cm, find the perimeter of ΔABC. **[Delhi-2000]**
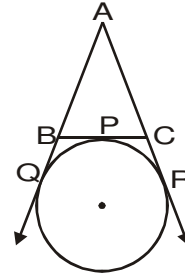
Q.6 A tangent PT is drawn parallel to a chord AB as shown in figure. Prove that APB is an isosceles triangle. **[Foreign-2000]**
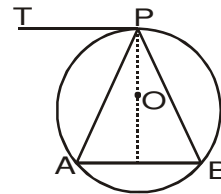
Q.7 In figure, XP and XQ are two tangents to a circle with centre O from a point X outside the circle. ARB is tangent to circle at R. Prove that XA + AR = XB + BR. **[Delhi-2003]**
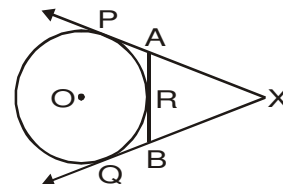
Q.8 In fig, if ∠ATO = 40°, find ∠AOB. **[AI-2008]**
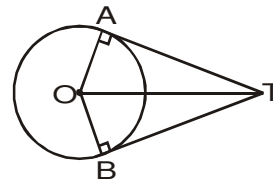
Q.9 In fig., CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm and BC = 7 cm, then find the length of BR. **[Delhi-2009]**

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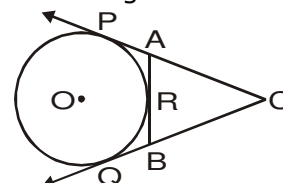
Q.10 In fig., ∆ABC is circumscribing a circle. Find the length of BC. **[AI-2009]**

Q.11 In fig., CP and CQ are tangents from an external point C to a circle with centre O. AB is another tangent which touches the circle at R. If CP = 11 cm and BR = 4 cm, find the length of BC. **[AI-2010]**
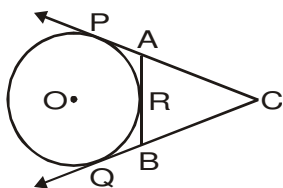
Q.12 If ∆ABC is isosceles with AB = AC, prove that the tangent at A to the circumcircle of ∆ABC is parallel to BC. **[AI-1998C]**

Q.13 In figure, AB and CD are two parallel tangents to a circle with centre O. ST is tangent segment between the two parallel tangents touching the circle at Q. Show that ∠SOT = 90°. **[AI-2000]**
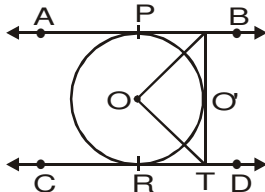
Q.14 A circle is inscribed in a ∆ABC having sides 8 cm, 10 cm and 12 cm as shown in figure. Find AD, BE and CF. **[Delhi-2001]**
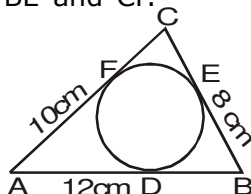
Q.15 PAQ is a tangent to the circle with centre O at a point A as shown in figure. If ∠OBA = 35°, find the value of ∠BAQ and ∠ACB.
[Foreign-2001]
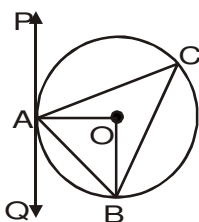
Q.16 AB is diameter and AC is a chord of a circle such that ∠BAC = 30°. If then tangent at C intersects AB produced in D, prove that BC = BD. **[Delhi-2003]**

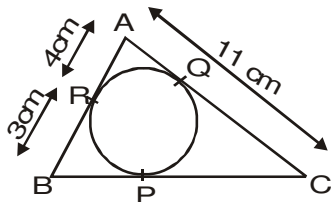**Q.17** ABC is an isosceles triangle in which AB = AC, circumscribed about a circle. Show that BC is bisected at the point of contact.

OR

In the fig., a circle is inscribed in a quadrilateral ABCD in which ∠B = 90°. If AD = 23 cm, AB = 29 cm and DS = 5 cm, find the radius (r) of the circle. **[Delhi-2008]**
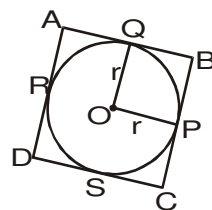
Q.18 In fig., OP is equal to diameter of the circle. Prove that ABP is an equilateral triangle. **[AI-2008]**
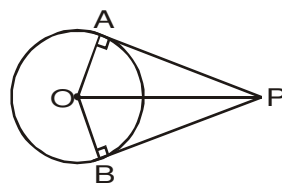
Q.19 Prove that a parallelogram circumscribing a circle is a rhombus. **[Foreign-2008]**

Q.20 Two tangents PA and PB are drawn to a circle with centre O from an external point P. Prove that ∠APB = 2∠OAB. **[Delhi-2009]**
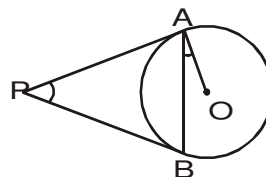
Q.21 In fig., a circle is inscribed in a triangle ABC having side BC = 8 cm, AC = 10 cm and AB = 12 cm. Find AD, BE and CF. **[Foreign-2009]**
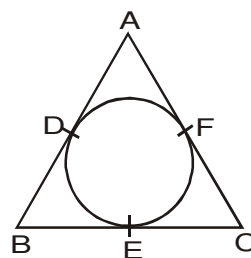
Q.22 In fig., there are two concentric circles with centre O and of radii 5 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, find the length of BP. **[AI-2010]**
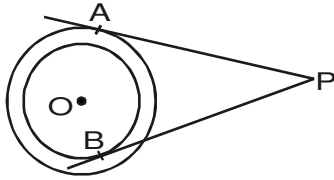
Q.23 Prove that the lengths of tangents drawn from an external point to a circle are equal. Using the above, prove the following :

A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC.

[Delhi-2008, AI-2009]
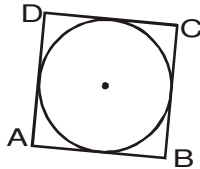
Q.24 Prove that the lengths of the tangents drawn from an external point to a circle are equal. Using the above, do the following:

In the fig., TP and TQ are tangents from T to the circle with centre O and R is any point on the circle. If AB is a tangent to the circle at R, prove that TA+AR = TB+BR. **[AI-2008]**
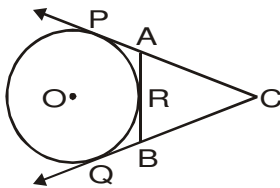
Q.25 Prove that the lengths of tangents drawn from an external point to a circle are equal. Using the above do the following :

ABC is an isosceles triangle in which AB = AC, circumscribed about a circle as shown in the fig. Prove that the base is bisected by the point of contact. **[Foreign-2008]**
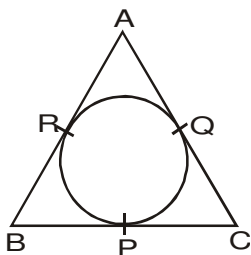
Q.26 Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. Using the above, do the following :

In fig., O is the centre of the two concentric circles. AB is a chord of the larger circle touching the smaller circle at C. Prove that AC = BC. **[AI-2009]**
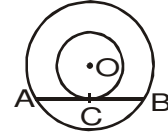
Q.27 Prove that the length of the tangents drawn from an external point to a circle are equal. Using the above, do the following :

In fig, quadrilateral ABCD is circumscribing a circle. Find the perimeter of the quadrilateral ABCD. **[Foreign-2009]**
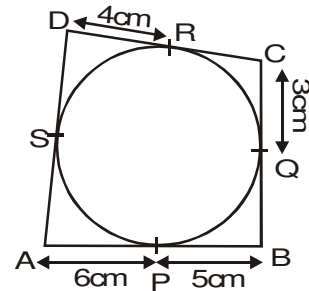
ANSWER KEY

|			
1.	x = 58°	**2.**	r = 2 cm
3.	PT = 6 cm	**4.**	32°, 26°
5.	10 cm	**8.**	100°
9.	4 cm	**10.**	10 cm
11.	7 cm	**14.**	7 cm, 5 cm, 3 cm
15.	55° and 55°	**17.**	OR 11 cm
21.	AD = 7 cm,	BE = 5 cm and CF = 3 cm	
22.	$4\sqrt{10}$ cm	**27.**	36 cm

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EXERCISE – III   MULTIPLE CHOICE QUESTIONS

Q.1 Which of the following statement is true ?
(A) The tangents drawn at the end points of a chord of a circle are parallel.
(B) Given a point P in the exterior of a circle, only two secants can be drawn through P to the circle.
(C) A secant of a circle intersects the circle in two distinct points.
(D) We can draw a tangent to a circle through its interior point.

Q.2 A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length of PQ is
(A) 12 cm   (B) 13 cm
(C) 8.5 cm   (D) $\sqrt{119}$ cm

Q.3 In the adjoining figure, PT is a tangent to a circle whose centre is O. If PT = 12 cm and radius of circle is 5 cm, then the distance of P from O is
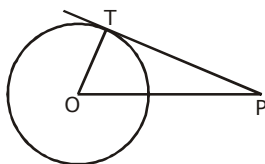
(A) $\sqrt{119}$ cm   (B) 13 cm
(C) 15 cm   (D) 17 cm

Q.4 In the adjoining figure, PQ is a tangent to a circle whose centre is C. If CP = 17 cm and the radius of the circle is 8 cm, then the length of the tangent PQ is
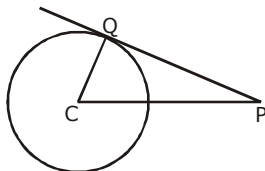
(A) 15 cm   (B) $\sqrt{353}$ cm
(C) 9 cm   (D) 12.5 cm

Q.5 rom a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is
(A) 7 cm   (B) 12 cm
(C) 15 cm   (D) 24.5 cm

Q.6 In the adjoining figure, TP and TQ are the two tangents to a circle with centre O. If ∠POQ = 110º, then ∠PTQ is

(A) 60º   (B) 70º
(C) 80º   (D) 90º
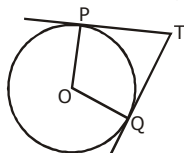
Q.7 In the adjoining figure, PA and PB are tangents from P to a circle with centre C. If ∠APB = 50º, then ∠ACB is

(A) 100º   (B) 110º
(C) 120º   (D) 130º
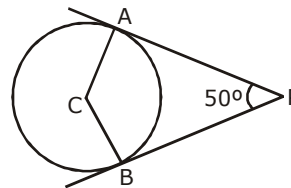
Q.8 In the adjoining figure, PQ and PR are tangents from P to a circle with centre O. If ∠POR = 55º, the ∠QPR is

(A) 35º   (B) 55º
(C) 70º   (D) 80º
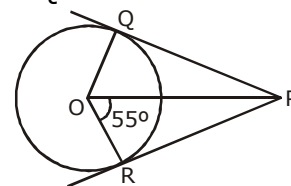
Q.9 If tangents PA and PB from an exterior point P to a circle with centre O are inclined to each other at an angle of 80°, then ∠POA is equal to
(A) 50°   (B) 60°
(C) 70°   (D) 100°

Q.10 In the adjoining figure, PA and PB are tangents from P to a circle with centre C. If the radius of the centre is 4 cm and PA ⊥ PB, then the length of each tangent is
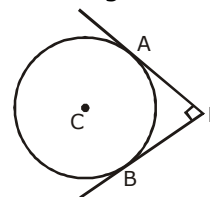
(A) 3 cm   (B) 4 cm
(C) 5 cm   (D) 6 cm

Q.11 If the length of a chord of a circle is 16 cm and is at a distance of 15 cm from the centre of the circle, then the radius of the circle (in cm) is –
(A) 15   (B) 16
(C) 17   (D) 34

Q.12 The radius of a circle is 6 cm. The perpendicular distance from the centre of the circle to the chord which is 8 cm in length, is –
(A) $\sqrt{5}$ cm   (B) $2\sqrt{5}$ cm
(C) $2\sqrt{7}$ cm   (D) $\sqrt{7}$ cm

Q.13 An equilateral triangle ABC is inscribed in a circle with centre O. Then, $\angle$ BOC is equal to–

(A) 30º   (B) 60º
(C) 90º   (D) 120º
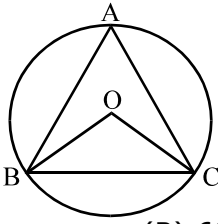
Q.14 In the adjoining figure, O is the centre of the circle. If $\angle$ OBC = 25º, then $\angle$ BAC is equal to

(A) 25º   (B) 30º
(C) 65º   (D) 150º
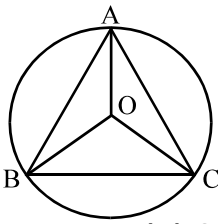
Q.15 In fig. O is the centre of the circle. If $\angle$ BAC = 52º, then $\angle$ OCD is equal to –

(A) 52º   (B) 104º
(C) 128º   (D) 76º
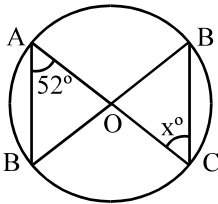
Q.16 In a circle with centre O, AB and CD are two diameters perpendicular to each other. The length of chord AC is –
(A) 2 AB   (B) $\sqrt{2}$ AB

(C) $\dfrac{1}{2}$ AB   (D) $\dfrac{1}{\sqrt{2}}$ AB

Q.17 In a circle with centre O, the unequal chords AB and CD intersect each other at P. Then, Δ APC and Δ DPB are –
(A) Equal in area   (B) Similar
(C) Congruent   (D) None of these

Q.18 In the given figure, AB and CD are two common tangents to the two touching circles. If DC = 4 cm, then AB is equal to –
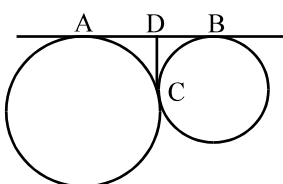
(A) 4 cm   (B) 6 cm
(C) 8 cm   (D) 12 cm

Q.19 CD is a direct common tangent to two circles intersecting each other at A and B. Then, $\angle$ CAD + $\angle$ CBD = ?

(A) 90º   (B) 180º
(C) 360º   (D) 120º
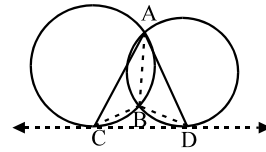
Q.20 In the adjoining figure, PQ is the tangent at K. If LN is a diameter and $\angle$ KLN = 30º, then $\angle$ PKL equals –

(A) 30º   (B) 50º
(C) 60º   (D) 70º
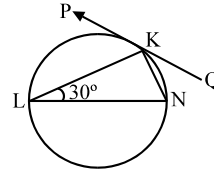
Q.21 In the adjoining figure, POQ is the diameter of the circle. R and S are any two points on the circle. Then,
(A) $\angle$ PRQ > $\angle$ PSQ
(B) $\angle$ PRQ < $\angle$ PSQ
(C) $\angle$ PRQ = $\angle$ PSQ

(D) $\angle$ PRQ = $\dfrac{1}{2}$ $\angle$ PSQ
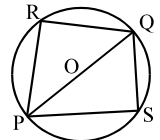
Q.22 Two equal circles of radius r intersect such that each passes through the centre of the other. The length of common chord is –
(A) $\sqrt{r}$ (B) $r\sqrt{2}$

(C) $r\sqrt{3}$ (D) $\dfrac{r\sqrt{3}}{2}$

Q.23 If four sides of a quadrilateral ABCD are tangential to a circle, then –
(A) AC + AD = BD + CD
(B) AB + CD = BC + AD
(C) AB + CD = AC + BC
(D) AC + AD = BC + DB

Q.24 In the adjoining figure, A, B, C are three points on a circle with centre O. If $\angle$ AOB = 90º and $\angle$ BOC = 120º, then $\angle$ ABC is–
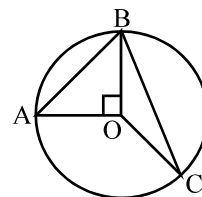
(A) 60º   (B) 75º
(C) 90º   (D) None of these

Q.25 AB is a diameter and AC is a chord of a circle such that $\angle$ BAC = 30º. The tangent at C intersects AB produced in D. Then,
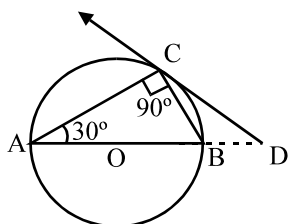
(A) BC < BD   (B) BC > BD
(C) BC = BD   (D) Cannot say

Q.26 The length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm, is –

(A) $\sqrt{7}$ cm   (B) $2\sqrt{7}$ cm
(C) 10 cm   (D) 5 cm

Q.27 Two circles of radii 20 cm and 37 cm intersect in A and B. If O and O' are their centres and AB = 24 cm, then distance OO' is equal to
(A) 45 cm   (B) 51 cm
(C) 40.5 cm   (D) 44 cm

Q.28 If two diameters of a circle intersect each other at right angles, then quadrilateral formed by joining their end points is a–
(A) Rhombus   (B) Rectangle
(C) Parallelogram   (D) Square

Q.29 If ABC is an arc of a circle and $\angle$ ABC = 135º, then the ratio of arc PQR to the circumference is
(A) 1 : 4   (B) 3 : 4
(C) 3 : 8   (D) 41 : 72

Q.30 If one angle of a cyclic trapezium is triple the other, then the greater one measures :
(A) 90º   (B) 105º
(C) 120º   (D) 135º

Q.31 The angle in a major segment of a circle is–
(A) Greater than 45º but less than 90º
(B) Less than 45º
(C) Less than 90º
(D) Greater than 90º but less than 135º

Q.32 O is the centre of a circle. If tangent PQ = 12 cm and BQ = 8 cm, then chord AB is –
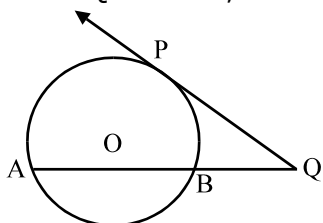
(A) 10 cm   (B) $4\sqrt{5}$ cm
(C) 4 cm   (D) 18 cm

Q.33 AB and CD are two parallel chords of a circle with centre O such that AB = 6 cm and CD = 12 cm. The chords are on the same side of the centre and the distance between them is 3 cm. Then, the radius of the circle is –
(A) 6 cm   (B) $5\sqrt{2}$ cm
(C) 7 cm   (D) $3\sqrt{5}$ cm

Q.34 In a circle of radius 17 cm, two parallel chords are drawn on opposite side of a diameter. The distance between the chords is 23 cm. if the length of one chord is 16 cm. Then the length of the other is –
(A) 34 cm   (B) 15 cm
(C) 23 cm   (D) 30 cm

Q.35 In the adjoining figure, $\angle$ ADC = 140º and AOB is the diameter of the circle. Then, $\angle$ BAC is equal to –

(A) 40º   (B) 50º
(C) 70º   (D) 75º
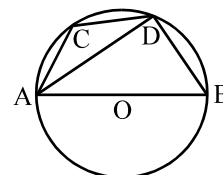
Q.36 If two circle are such that the centre of one lies on the circumference of the other, then the ratio of the common chord of the two circles to the radius of any one of the circles, is –
(A) 4 : 1   (B) $\sqrt{5}$: 1
(C) 2 : 1   (D) $\sqrt{3}$: 1

Q.37 If tangents QR, RP, PQ are drawn respectively at A, B, C to a circle circumscribing an acute angled Δ ABC so as to form another Δ PQR, then, $\angle$ RPQ is equal to –

(A) 180º – $\angle$ BAC   (B) $\dfrac{1}{2}$(180º – $\angle$ BAC)
(C) $\angle$ BAC   (D) 180º – 2 $\angle$ BAC

Q.38 Two circles touch externally. The sum of their areas is 130 π sq cm and the distance between their centres is 14 cm. The radius of the smaller circle is
(A) 2 cm   (B) 3 cm
(C) 4 cm   (D) 5 cm

Q.39 Two circles touch internally. The sum of their areas is, 116 π sq. cm and the distance between their centres is 6 cm. The radius of the larger circle is –
(A) 7 cm   (B) 8 cm
(C) 10 cm   (D) 12 cm

Q.40 Two circles touch each other internally. Their radii are 2 cm and 3 cm. The biggest chord of the outer circle which is outside the inner circle, is of length
(A) $2\sqrt{2}$ cm   (B) $3\sqrt{2}$ cm
(C) $2\sqrt{3}$ cm   (D) $4\sqrt{2}$ cm

Q.41 The chord of a circle is equal to its radius. The angle subtended by this chord at the minor arc of the circle is
(A) 60º   (B) 75º
(C) 120º   (D) 150º

Q.42 The distance between the centres of the two circles of radii r_1 and r_2 is d. They will touch each other internally if
(A) d = r_1 or r_2 (B) d = r_1 + r_2
(C) d = r_1 – r_2 (D) d = $\sqrt{r_1 r_2}$

Q.43 A triangle with sides 13 cm, 14 cm and 15 cm is inscribed in a circle. The radius of the circle is –
(A) 2 cm   (B) 3 cm
(C) 4 cm   (D) 5 cm

Q.44 Two chords AB and CD of a circle intersect at E such that AE = 2.4 cm, BE = 3.2 cm and CE = 1.6 cm. the length of DE is –
(A) 1.6 cm   (B) 3.2 cm
(C) 4.8 cm   (D 6.4 cm

Q.45 A chord of length 14 cm is at a distance of 6 cm from the centre of a circle. The length of another chord at a distance of 2 cm from the centre of the circle is –
(A) 12 cm   (B) 14 cm
(C) 16 cm   (D) 18 cm

Q.46 The number of common tangents that can be drawn to two given circles is at the most:
(A) One   (B) Two
(C) Three   (D) Four

Q.47 A is a point outside the circle with centre O, AT and AT' are the tangents to the circle and P is a point on the circle as shown in the figure. Then, ∠ TPT' equals –

(A) 55º   (B) 70º
(C) 125º   (D) 140º
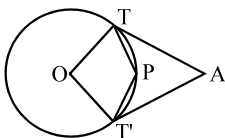
Q.48 Two circles of radii R and r touch each other externally and PQ is the direct common tangent. Then PQ^2 is equal to
(A) R – r   (B) R + r
(C) 2 r R   (D) 4 r R

Q.49 If O is the centre of a circle of radius r and AB is a chord of the circle at a distance $\frac{r}{2}$ from O, then ∠ BAO is –

(A) 60º   (B) 45º
(C) 30º   (D) 15º
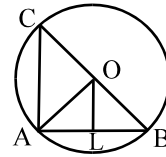
Q.50 ABC is a right angled triangle with BC = 6 cm and AB = 8 cm. A circle with centre O is inscribed in Δ ABC. The radius of the circle is –
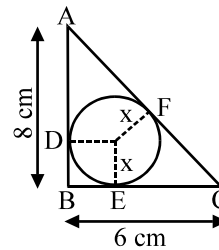
(A) 1 cm   (B) 2 cm
(C) 3 cm   (D) 4 cm

Q.51 In the adjoining figure AD, AE and BC are tangents to the circle at D, E, F respectively. Then –
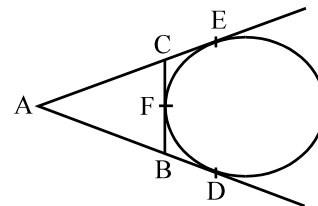
(A) AD = AB + BC + CA
(B) 2 AD = AB + BC + CA
(C) 3 AD = AB + BC + CA
(D) 4 AD = AB + BC + CA

Q.52 In the adjoining figure, the larger circle with radius 4 cms is touched internally by two smaller circles which also touch each other externally at the centre O of the larger circle. The area of shaded portion is –

(A) 4 π (B) 7 π
(C) 12 π (D) 16 π
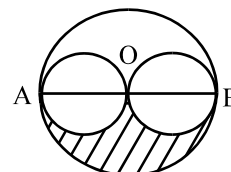
Q.53 In the adjoining figure, a smaller circle touches a larger circle internally and passes through the centre O of the larger circle. If the area of the smaller circle. If the area of te smaller circle is 200 cm^2, the area of the larger circle in sq. cm is –
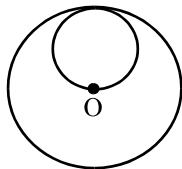
(A) 400   (B) 600
(C) 800   (D) 1000

Q.54 ACB is a tangent to a circle at C. CD and CE are chords such that $\angle$ ACE > $\angle$ ACD. If $\angle$ ACD = $\angle$ BCE = 50º, then
(A) CD = CE
(B) ED is not parallel to AB
(C) ED passes through the centre of the circle
(D) Δ CDE is a right angled triangle.

Q.55 AB is a chord of a circle whose centre is O. P is a point on the circle such that OP $\perp$ AB and OP intersect AB at the point M. If AB = 8 cm and MP = 2 cm, then the radius of the circle is –
(A) 10 cm   (B) 6 cm
(C) 5 cm   (D) 4 cm

Q.56 In the adjoining figure, PQ is a tangent from P to the circle and QOR is a diameter. If $\angle$ POR = 130º, $\angle$ QPO is –

(A) 40º   (B) 45º
(C) 50º   (D) 75º
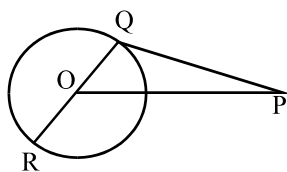
Q.57 Two circles touch each other externally at C and AB is a common tangent to the circles. Then, $\angle$ ACB is –
(A) Equal to 90º
(B) Less than 90º
(C) Greater than 90º
(D) Greater than 120º

Q.58 If two equal circles touch each other externally, the common tangent divides the line of centres in the ratio –
(A) 1 : 1   (B) 2 : 1
(C) 1 : 2   (D) 3 : 2

Q.59 Three equal circles of unit radius touch each other. Then, the area of the circle circumscribing the three circles is –

(A) $\frac{\pi}{3}(2+\sqrt{3})^2$ (B) $6\pi(2+\sqrt{3})^2$

(C) $\frac{1}{3\pi}(2+\sqrt{3})^2$ (D) $\frac{1}{6}\pi(2+\sqrt{3})^2$

Q.60 Three points A, B, C are on the same line. A circle passes Through B and C. Then the focus of the tangent drawn from A to the circle, if the diameter of the circle is 2a, is–
(A) $x^2 + y^2 = a^2$ (B) $xx_1 + yy_1 = a^2$
(C) xy = 0   (D) x + y = 0

Q.61 AC is tangent to a circle with centre O at the point A. Δ OAC is an isosceles triangle. $\angle$ OCA is equal to –
(A) 30º   (B) 45º
(C) 60º   (D) 90º

Q.62 Two circles of radii 8 cm and 5 cm are drawn with centres O and O' respectively. Their transverse common tangents meet OO' in A. The point A divides OO' in the ratio –
(A) 8 : 5 internally
(B) 5 : 8 internally
(C) 5 : 8 externally
(D) 8 : 5 externally

**Q.63** M and N are the centres of two circles whose radii are 7 cm and 4 cm respectively. The direct common tangents to the circles meet MN in P. Then, P divides MN in the ratio –
(A) 7 : 4 internally   (B) 4 : 7 internally
(C) 7 : 4 externally (D) 4 : 7 externally

Q.64 The radius of a circle is 20 cm. The radii (in cm) of three concentric circles drawn in such a manner that the whole area is divided into four equal parts, are –

(A) $20\sqrt{2}, 20\sqrt{3}, 20$ (B) $\frac{10\sqrt{3}}{3}, \frac{10\sqrt{2}}{3}, \frac{10}{3}$

(C) $10\sqrt{3}, 10\sqrt{2}, 10$ (D) 17, 14, 10

Q.65 PQR is a right angled triangle with hypotenuse PR. A circle with centre O is inscribed in the triangle. If PQ = 6 cm and QR = 8 cm, then the radius of the circle is –
(A) 1.8 cm   (B) 2 cm
(C) 2.5 cm   (D) 3.6 cm

Q.66 The chord AB of a circle of centre O subtends an angle θ with the tangent at A to the circle. $\angle$ ABO is –
(A) θ (B) $2 \times (\pi - \theta)$
(C) 90º – θ (D) 90º + θ

Q.67 If three equal circles of radius 3 cm each touch each other, then the area of the shaded portion is –
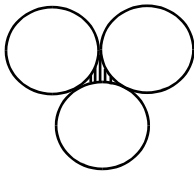
(A) $\dfrac{\sqrt{3}}{2}(2-\pi)\,cm^2$ (B) $\dfrac{9}{2}(2\sqrt{3}-\pi)\,cm^2$

(C) $\dfrac{9}{2}(2\sqrt{3}+\pi)\,cm^2$ (D) $\dfrac{3}{2}(\sqrt{3}-\pi)\,cm^2$

Q.68 Any cyclic parallelogram having unequal adjacent sides is necessarily a –
(A) Square   (B) Rectangle
(C) Rhombus   (D) Trapezium

Q.69 A cyclic quadrilateral whose opposite angles are equal, is a –
(A) Parallelogram but not a rhombus
(B) Rhombus
(C) Rectangle
(D) Square but not a rectangle

Q.70 The radius of the circumscribing circle of an equilateral triangle of side 6 cm, is –
(A) $2\sqrt{3}$ cm   (B) $3\sqrt{3}$ cm

(C) $\sqrt{33}$ cm   (D) $4\sqrt{3}$ cm

Q.71 AD is a diameter of a circle and AB is a chord. If AD = 34cm, AB = 30cm, the distance of AB from the centre of the circle is: **[NTSE]**
(A) 17cm   (B) 15cm
(C) 4cm   (D) 8cm

Q.72 In figure, if OA = 5cm, AB = 8cm and OD is perpendicular to AB, then CD is equal to: **[NTSE]**
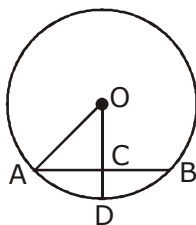
(A) 2cm   (B) 3cm
(C) 4cm   (D) 5cm

Q.73 If AB = 12cm, BC = 16cm and AB is perpendicual to BC, then the radius of the circle passing through the points A, B and C is: **[NTSE]**
(A) 6cm   (B) 8cm
(C) 10cm   (D) 12cm

Q.74 In figure, if ∠ABC = 20°, then ∠AOC is equal to: **[NTSE]**
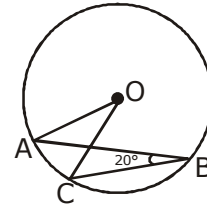
(A) 20°   (B) 40°
(C) 60°   (D) 10°

Q.75 In figure, if AOB is a diameter of the circle and AC = BC, then ∠CAB is equal to: **[NTSE]**
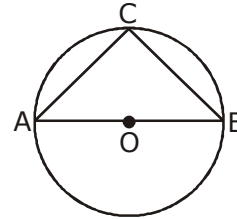
(A) 30°   (B) 60°
(C) 90°   (D) 45°

ANSWER KEY

1.	C	**2.**	D	**3.**	B	**4.**	A
5.	A	**6.**	B	**7.**	D	**8.**	C
9.	A	**10.**	B	**11.**	C	**12.**	B
13.	D	**14.**	C	**15.**	A	**16.**	D
17.	B	**18.**	C	**19.**	B	**20.**	C
21.	C	**22.**	C	**23.**	B	**24.**	B
25.	C	**26.**	C	**27.**	B	**28.**	D
29.	C	**30.**	D	**31.**	C	**32.**	A
33.	D	**34.**	D	**35.**	B	**36.**	D
37.	D	**38.**	B	**39.**	C	**40.**	D
41.	D	**42.**	C	**43.**	C	**44.**	C
45.	D	**46.**	D	**47.**	C	**48.**	D
49.	C	**50.**	B	**51.**	B	**52.**	B
53.	C	**54.**	A	**55.**	C	**56.**	A
57.	D	**58.**	A	**59.**	A	**60.**	B
61.	B	**62.**	A	**63.**	C	**64.**	C
65.	B	**66.**	C	**67.**	B	**68.**	D
69.	C	**70.**	A	**71.**	D	**72.**	A
73.	C	**74.**	B	**75.**	D		

EXERCISE – IV   NTSE /OLYMPIAD /FOUNDATION PROBLEMS

CHOOSE THE CORRECT ONE

1. If the diagonals of a cyclic quadrilateral are equal, then the quadrilateral is

(A) rhombus (B) square (C) rectangle (D) none of these

2. The quadrilateral formed by angle bisectors of a cyclic quadrilateral is a:

(A) rectangle (B) square (C) parallelogram (D) cyclic quadrilateral

3. In the given figure, AB is the diameter of the circle. Find the value of $\angle$ACD :

(A) 30°

(B) 60°

(C) 45°

(D) 25°
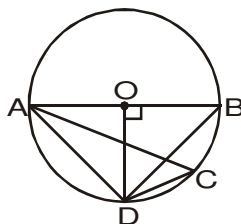
4. Find the value of $\angle$DCE :

(A) 100°

(B) 80°

(C) 90°

(D) 75°
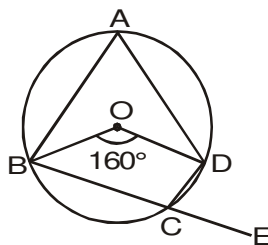
5. In the given figure, PQ is the tangent of the circle. Line segment PR intersects the circle at Nand R. PQ = 15 cm, PR = 25 cm, find PN:

(A) 15 cm

(B) 10 cm

(C) 9 cm

(D) 6 cm
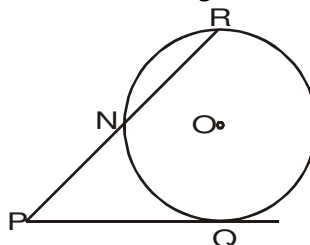
6. In the given figure, there are two circles with the centres O and O' touching each other internally at P. Tangents TQ and TP are drawn to the larger circle and tangents TP and TR are drawn to the smaller circle. Find TQ : TR

(A) 8 : 7

(B) 7 : 8

(C) 5 : 4

(D) 1 : 1
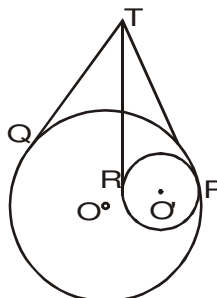
7. In the given figure, PAQ is the tangent. BC is the diameter of the circle. m $\angle$BAQ = 60°, find m $\angle$ABC :

(A) 25°

(B) 30°

(C) 45°

(D) 60°
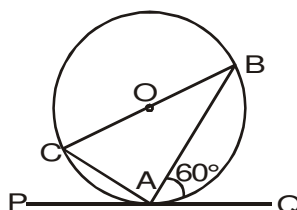
8. ABCD is a cyclic quadrilateral PQ is a tangent at B. If ∠DBQ = 65° , then ∠BCD is :

(A) 35°

(B) 85°

(C) 115°

(D) 90°
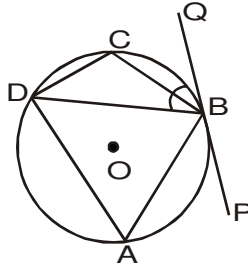
9. In the given figure, AP = 2 cm, BP = 6 cm and CP = 3 cm. Find DP :

(A) 6 cm

(B) 4 cm

(C) 2 cm

(D) 3 cm
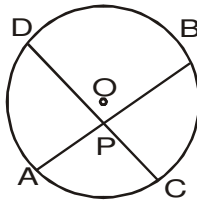
10. In the given figure, AP = 3 cm, BA = 5 cm and CP = 2 cm. Find CD :

(A) 12 cm

(B) 10 cm

(C) 9 cm

(D) 6 cm
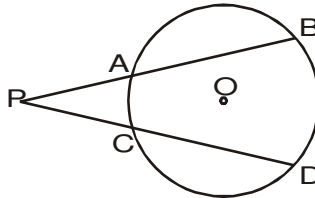
11. In the given figure, tangent PT = 5 cm, PA = 4 cm, find AB :

(A) $\dfrac{7}{4}$ cm

(B) $\dfrac{11}{4}$ cm

(C) $\dfrac{9}{4}$ cm

(D) can't be determined
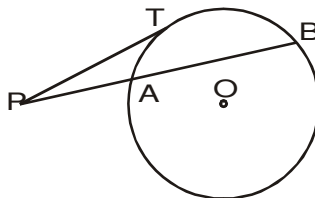
12. Two circles of radii 13 cm and 5 cm touch internally each other. Find the distance between their centres

(A) 18 cm   (B) 12 cm   (C) 9 cm   (D) 8 cm

13. Three circles touch each other externally. The distance between their centre is 5 cm, 6 cm and 7 cm. Find the radii of the circles :

(A) 2 cm, 3 cm, 4 cm   (B) 3 cm, 4 cm, 1 cm

(C) 1 cm, 2.5 cm, 3.5 cm   (D) 1 cm, 2 cm, 4 cm

14. If AB is a chord of a circle, P and Q are two points on the circle different from A and B, then:

(A) the angle subtended by AB at P and Q are either equal or supplementary.

(B) the sum of the angles subtended by AB at P and Q is always equal two right angles.

(C) the angles subtended at P and Q by AB are always equal.

(D) the sum of the angles subtended at P and Q is equal to four right angles.

15. In the given figure, CD is a direct common tangent to two circles intersecting each other at A and B, then:

∠CAD + ∠CBD = ?

(A) 120°

(B) 90°

(C) 360°

(D) 180°
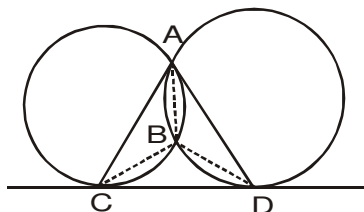
16. In a circle of radius 5 cm, AB and AC are the two chords such that AB = AC = 6 cm. Find the length of the chord BC.

(A) 4.8 cm   (B) 10.8 cm   (C) 9.6 cm   (D) none of these

17. In a circle of radius 17 cm, two parallel chords are drawn on opposite sides of a diameter. The distance between the chords is 23 cm. If the length of one chord is 16 cm, then the length of the other is :

(A) 23 cm   (B) 30 cm   (C) 15 cm   (D) none of these

18. If two circles are such that the centre of one lies on the circumference of the other, then the ratio of the common chord of two circles to the radius of any of the circles is :

(A) $\sqrt{3} : 2$ (B) $\sqrt{3} : 1$ (C) $\sqrt{5} : 1$ (D) none of these

19. Two circles touch each other internally. Their radii are 2 cm and 3 cm. The biggest chord of the other circle which is outside the inner circle, is of length :

(A) $2\sqrt{2}$ cm   (B) $3\sqrt{2}$ cm   (C) $2\sqrt{3}$ cm   (D) $4\sqrt{2}$ cm

20. Through any given set of four points P,Q, R, S it is possible to draw :

(A) atmost one circle (B) exactly one circle (C) exactly two circles   (D) exactly three circles

21. The distance between the centres of equal circles each of radius 3 cm is 10 cm. The length of a transverse tangent is :

(A) 4 cm   (B) 6 cm   (C) 8 cm   (D) 10 cm

22. The number of common tangents that can be drawn to two given circles is at the most :

(A) 1   (B) 2   (C) 3   (D) 4

23. ABC is a right angled triangle AB = 3 cm, BC = 5 cm and AC = 4 cm, then the inradius of the circle is :

(A) 1 cm

(B) 1.25 cm

(C) 1.5 cm

(D) none of these

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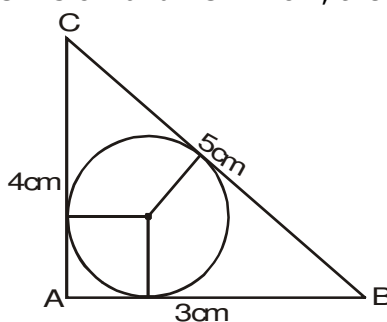
24. A circle has two parallel chords of lengths 6 cm and 8 cm. If the chords are 1 cm apart and the centre is on the same side of the chords, then a diameter of the circle is of length:

(A) 5 cm   (B) 6 cm   (C) 8 cm   (D) 10 cm

25. In the adjoining figure AB is a diameter of the circle and $\angle BCD = 130°$. What is the value of $\angle ABD$?

(A) 30°

(B) 50°

(C) 40°

(D) None of these
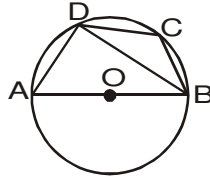
26. In the given figure O is the centre of the circle and $\angle BAC = 25°$, then the value of $\angle ADB$ is :

(A) 40°

(B) 55°

(C) 50°

(D) 65°
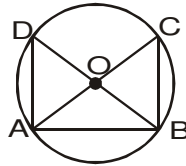
27. In the given circle O is the centre of the circle and AD, AE are the two tangents. BC is also a tangent, then :

(A) AC + AB = BC

(B) 3AE = AB + BC + AC

(C) AB + BC + AC = 4AE

(D) 2AE = AB + BC + AC
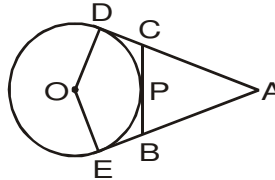
28. In a circle O is the centre and $\angle COD$ is right angle. AC = BD and CD is the tangent at P. What is the value of AC + CP, if the radius of the circle is 1 metre?

(A) 105 cm

(B) 141.4 cm

(C) 138.6 cm

(D) Can't be determined
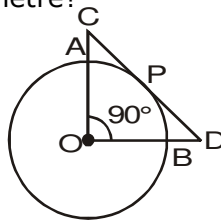
29. In a triangle ABC, O is the centre of incircle PQR, $\angle BAC = 65°$, $\angle BCA = 75°$, find $\angle ROQ$:

(A) 80°

(B) 120°

(C) 140°

(D) Can't be determined
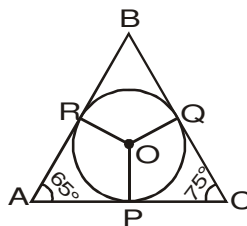
30. In the adjoining figure O is the centre of the circle. $\angle AOD = 120°$. If the radius of the circle be 'r', then find the sum of the areas of quadrilaterals AODP and OBQC :

(A) $\dfrac{\sqrt{3}}{2}r^2$

(B) $3\sqrt{3}r^2$

(C) $\sqrt{3}r^2$

(D) None of these

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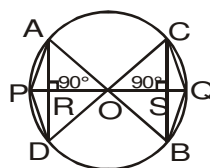
31. There are two circles each with radius 5 cm. Tangent AB is 26 cm. The length of tangent CD is :

(A) 15 cm

(B) 21 cm

(C) 24 cm

(D) Can't be determined
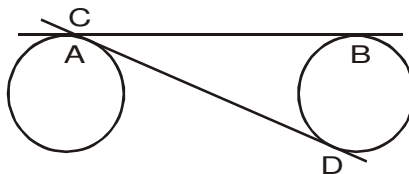
32. In the adjoining figure O is the centre of the circle and AB is the diameter. Tangent PQ touches the circle at D. ∠BDQ = 48°. Find the ratio of ∠DBA : ∠DCB :

(A) $\dfrac{22}{7}$

(B) $\dfrac{7}{22}$

(C) $\dfrac{7}{12}$

(D) Can't be determined
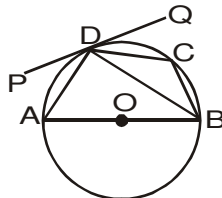
33. In the given diagram O is the centre of the circle and CD is a tangent. ∠CAB and ∠ACD are supplementary to each other ∠OAC = 30°. Find the value of ∠OCB :

(A) 30°

(B) 20°

(C) 60°

(D) None of these
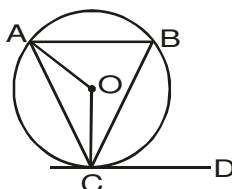
34. In the given diagram an incircle DEF is circumscribed by the right angled triangle in which AF = 6 cm and EC = 15 cm. Find the difference between CD and BD :

(A) 1 cm

(B) 3 cm

(C) 4 cm

(D) Can't be determined
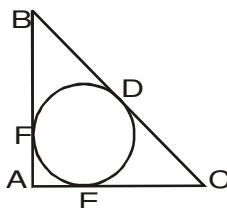
35. In the adjoining figure 'O' is the centre of circle, ∠CAO = 25° and ∠CBO = 35°. What is the value of ∠AOB?

(A) 55°

(B) 110°

(C) 120°

(D) Data insufficient
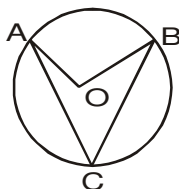
36. In the given figure 'O' is the centre of the circle SP and TP are the two tangents at S and T respectively. ∠SPT is 50°, the value of ∠SQT is :

(A) 125°

(B) 65°

(C) 115°

(D) None of the above
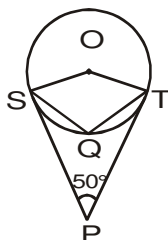
37. In the given figure of circle, 'O' is the centre of the circle ∠AOB = 130°. What is the value of ∠DMC?

(A) 65°

(B) 125°

(C) 85°

(D) Can't be determined
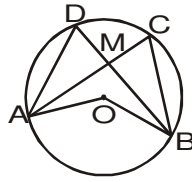
38. In the adjoining figure 'O' is the centre of the circle and PQ, PR and ST are the three tangents. ∠QPR = 50°, then the value of ∠SOT is :

(A) 30°

(B) 75°

(C) 65°

(D) Can't be determined
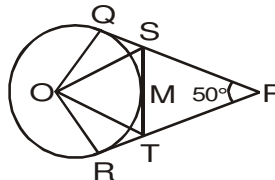
39. ABC is an isosceles triangle and AC, BC are the tangents at M and N respectively. DE is the diameter of the circle. ∠ADP = ∠BEQ = 100°. What is value of ∠PRD?

(A) 60°

(B) 50°

(C) 20°

(D) Can't be determined

40. In the adjoining figure the diameter of the larger circle is 10 cm and the smaller circle touches internally the lager circle at P and passes through O, the centre of the larger circle. Chord SP cuts the smaller circle at R and OR is equal to 4 cm. What is the length of the chord SP?

(A) 9 cm

(B) 12 cm

(C) 6 cm

(D) $8\sqrt{2}$ cm
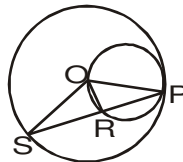
41. In the given figure ABCD is a cyclic quadrilateral DO = 8 cm and CO = 4 cm. AC is the angle bisector of ∠BAD. The length of AD is equal to the length of AB. DB intersects diagonal AC at O, then what is the length of the diagonal AC?

(A) 20 cm

(B) 24 cm

(C) 16 cm

(D) None of these
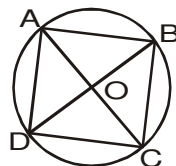
OBJECTIVE						ANSWER KEY							EXERCISE -4		
Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	D	C	B	C	D	B	C	B	B	C	D	A	A	D
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	C	B	B	D	A	C	B	A	D	C	D	D	B	C	C
Que.	31	32	33	34	35	36	37	38	39	40	41				
Ans.	C	B	A	A	C	C	D	C	C	C	A				

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CONSTRUCTIONS

INTRODUCTION

In class IX, we have discussed a number of constructions with the help of ruler and compass e.g. bisecting a line segment, bisecting an angle, perpendicular bisector of line segment, some more constructions of triangles etc. with their justifications. In this chapter we will discuss more constructions by using the knowledge of the earlier construction.

DIVISION OF A LINE SEGMENT

Let us divide the given line segment in the given ratio say 5 : 8. This can be done in the following two ways:

(i)   Use of Basic Proportionality Theorem.

(ii)   Constructing a triangle similar to a given triangle.

BASIC CONCEPTS AND IMPORTANT RESULTS

In this chapter, you will do the following constructions. Students are expected to know the mathematical reasons. why such constructions work.

1. *To divide a line segment in a given ratio.*

2. *To construct a triangle similar to a given triangle as per given scale factor.*

   Scale factor means the ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle.

   This construction involves two different situations.

   (i) The triangle to be constructed is smaller than the given triangle, here scale factor is less than 1.

   (ii) The triangle to be constructed is bigger than the given triangle, here scale factor is greater than 1.

3. *To construct a tangent to a circle at a given point in it* (*using the centre of the circle*).

4. *To construct two tangents to a circle from a point outside the circle* (*using the centre of the circle*)..

   If the centre of a circle is not given, you may locate it by taking any two non-parallel chords of the circle and then finding the point of intersection of their perpendicular bisectors.

   Divide a line segment 6 cm long in the ratio 4 : 3. Prove your assertion.

STEPS OF CONSTRUCTION

Step 1. Draw a line segment AB = 6 cm.

Step 2. Draw a ray AX, making an acute angle $\angle BAX$.

Step 3. Along AX, mark (4 + 3) = 7 points A_1, A_2, A_3, A_4, A_5, A_6 and A_7 such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$

Step 4. Join A_7B.

Step 5. From A_4, draw A_4C || A_7B, meeting AB at C.

   Then, C is the point on AB, which divides it in the ratio 4 : 3.
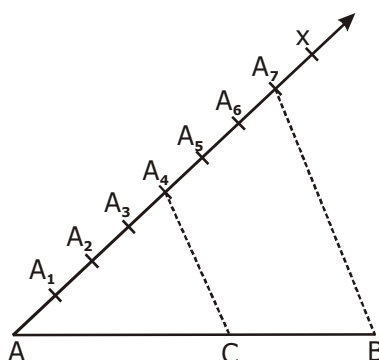
Thus, AC : CB = 4 : 3.

Proof : Let $AA_1 = A_1A_2 = ... = A_6A_7 = x.$

In $\triangle ABA_7$, we have $A_4C \parallel A_7B.$

Hence, AC : CB = 4 : 3.

Construction-1: Draw a segment of length 7.6 cm and divide it in the ratio 5 : 8. Measure the two parts.
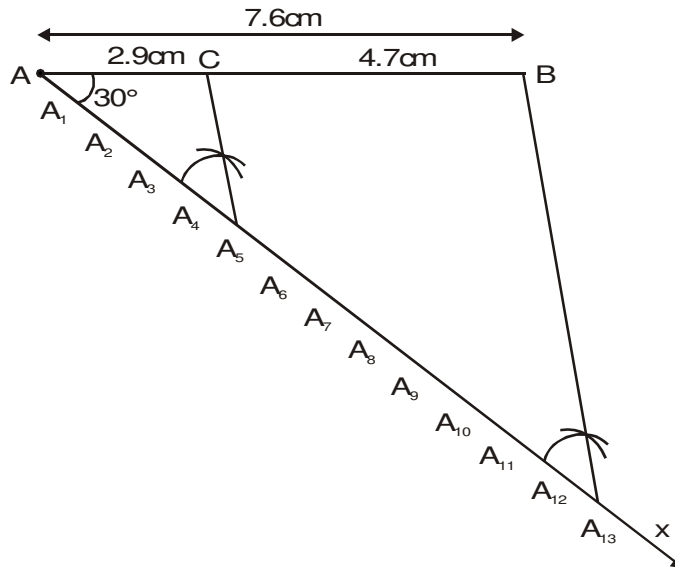
STEPS OF CONSTRUCTION :

Step 1 : Draw any ray AX making an angle of 30° with AB.

Step 2 : Locate 13 points : A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}, A_{11}, A_{12} and A_{13} So that: $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4 A_5 = ... = A_{11} A_{12} = A_{12} A_{13}$

Step 3 : Join B with A_{13}.

Step 4 : Through the point A_5, draw a line $A_5C \parallel A_{13}B$ such that $\angle AA_5C$ = corr. $\angle AA_{13}B$ intersecting AB at a point C. Then AC : CB = 5 : 8.

Let us see how this method gives us the required division.

Since A_5C is parallel to $A_{13} B.$

Therefore $\dfrac{AA_5}{A_5A_{13}} = \dfrac{AC}{CB}$ (Basic Proportionality Theorem)

By construction, $\dfrac{AA_5}{A_5A_{13}} = \dfrac{5}{8}$

Therefore $\dfrac{AC}{CB} = \dfrac{5}{8}$

This given that C divides AB in the ratio 5 : 8.

By measurement, we find, AC = 2.9 cm, CB = 4.7 cm.

By Calculation: $AC = \dfrac{7.6 \times 5}{13} = \dfrac{38}{13}$ = 2.9 cm

$BC = \dfrac{7.6 \times 8}{13} = \dfrac{60.8}{13}$ = 4.67 = 4.7 cm.

ALTERNATIVE SOLUTION

Step 1 : Draw a line segment AB = 7.6 cm and to be divided in the ratio 5 : 8.

Step 2 : Draw any ray AX making an angle of 30° with AB.

Step 3 : Draw a ray BY parallel to AX by making ∠ABY equal to ∠BAX. i.e. ∠ABY = corr. ∠BAX.
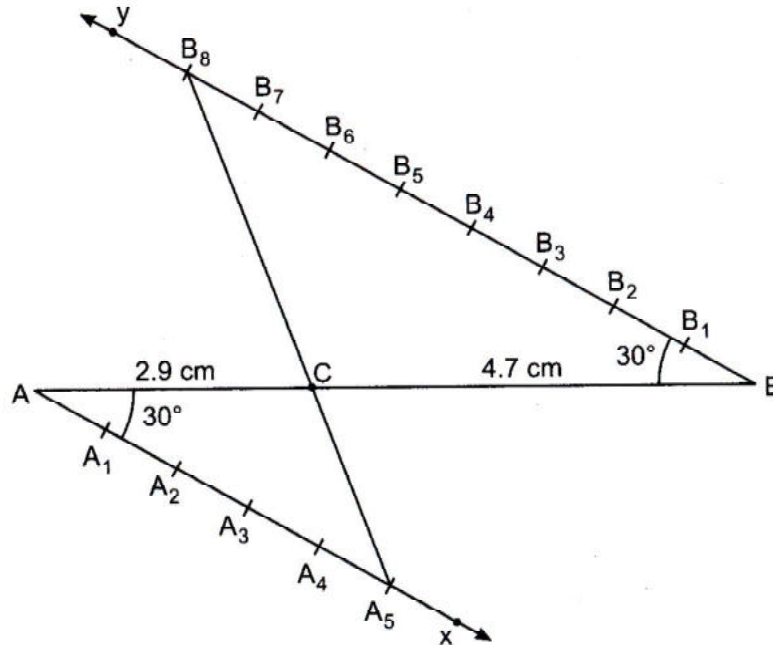
Step 4 : Locate the points A_1, A_2, A_3, A_3, A_4, A_5 on AX and B_1, B_2, B_3, B_4, B_5, B_6, B_7, and B_8 on BY such that: AA_1 = A_1A_2 = ... = A_4A_5 = BB_1 = B_1B_2 = ... = B_6B_7 = B_7B_8.

Step 5 : Join A_5B_8. Let it intersect AB at a point C. Then AC : CB = 5 : 8.

Here $\triangle AA_5C$ is similar to $\triangle BB_8C$

Then $\qquad \dfrac{AA_5}{BB_8} = \dfrac{AC}{BC}$

Since by construction, $\qquad \dfrac{AA_5}{BB_8} = \dfrac{5}{8}$

Therefore $\dfrac{AC}{CB} = \dfrac{5}{8}$

By measurement : AC = 2.9 cm, BC = 4.7 cm.

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SOLVED PROBLEMS

Ex.1 Construct a triangle with sides 5 cm, 6 cm and 7 cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle. **[NCERT]**

Sol. First of all we are to construct a triangle ABC with given sides, AB = 6 cm, BC = 7 cm, CA = 5 cm. Given a triangle ABC, we are required to construct a triangle whose sides are $\frac{7}{5}$ of the corresponding sides of ∆ABC.

STEPS OF CONSTRUCTION :

Step 1 : Draw any ray BX making an angle of 30° with the base BC of ∆ABC on the opposite side of the vertex A.

Step 2 : Locate seven points B_1, B_2, B_3, B_4, B_5, B_6 and B_7 on BX so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7$. [Note that the number of points should be greater of m and n in the scale factor $\frac{m}{n}$.]
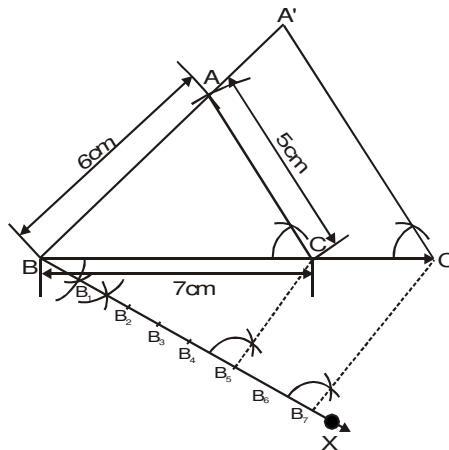
Step 3 : Join B_5 (the fifth point) to C and draw a line through B_7 parallel to B_5C, intersecting the extended line segment BC at C'.

Step 4 : Draw a line through C' parallel to CA intersecting the extended line segment BA at A'.

Then, A'BC' is the required triangle.

For justification of the construction.

$$\Delta ABC \sim \Delta A'BC'$$

Therefore, $\qquad \dfrac{AB}{A'B} = \dfrac{AC}{A'C} = \dfrac{BC}{BC'}$

But $\qquad \dfrac{BC}{BC'} = \dfrac{BB_5}{BB_7} = \dfrac{5}{7}$

Therefore $\dfrac{A'B}{AB} = \dfrac{A'C}{AC} = \dfrac{BC'}{BC} = \dfrac{7}{5}$

Ex.2 Draw a triangle ABC with side BC = 6 cm, AB = 5 cm and ∠ABC = 60°. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC. **[NCERT]**

Sol. Given a triangle ABC, we are required to construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.
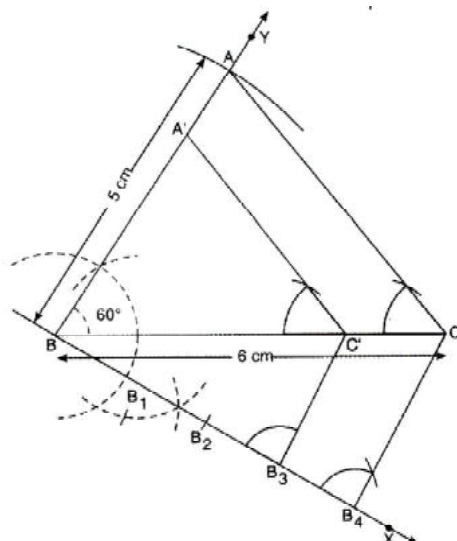
Steps of Construction :

Step 1 : Draw a line segment BC = 6 cm.

Step 2 : At B construct $\angle CBY = 60°$ and cut off AB = 5 cm, join AB and AC. ABC is the required $\triangle$.

Step 3 : Draw any ray BX making an acute angle say 30° with BC on the opposite side of the vertex A, $\angle CBX = 30°$ downwards.

Step 4 : Locate four (the greater of 3 and 4 in $\frac{3}{4}$) points B_1, B_2, B_3 and B_4 on BX, so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

Step 5 : Join B_4C and draw a line through B_3 (the 3rd point) parallel to B_4C to intersect BC at C'.

Step 6 : Draw a line through C' parallel to the line CA to intersect BA at A'.

Then A'BC' is the required triangle whose each side is $\frac{3}{4}$ times the corresponding sides of the $\triangle$ABC. Let us now see how this construction gives the required triangle.

For justification of the construction.

$$\frac{BC'}{C'C} = \frac{3}{1}$$

Therefore $\quad \frac{BC}{BC'} = \frac{BC'+C'C}{BC'} = \frac{BC'}{BC'} + \frac{C'C}{BC'} = 1 + \frac{1}{3} = \frac{4}{3}$

$\Rightarrow \quad BC' = \frac{3}{4} BC$, Also C'A' is parallel to CA.

Therefore $\triangle A'BC' \sim \triangle ABC$

$\Rightarrow \quad \frac{A'B}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC} = \frac{3}{4}$

★ **CONSTRUCTION OF TANGENTS TO A CIRCLE**

(a) If a point lies inside a circle, we can not draw any tangent to the circle i.e., No tangent is possible in this case.

(b) If a point lies on the circle, then there is only one tangent to the circle at this point. The tangent to a circle at any point is perpendicular to the radius passing through the point of contact.
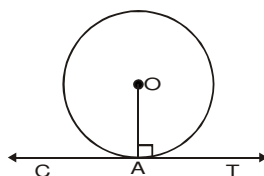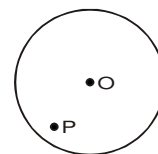
(c) Two tangents are drawn from an external point to circle, they are equal in length.
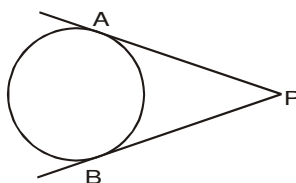
Ex.3 Draw a circle of radius 5 cm. From a point 8 cm away from its centre, construct pair of tangents to the circle measure their lengths.

Sol.
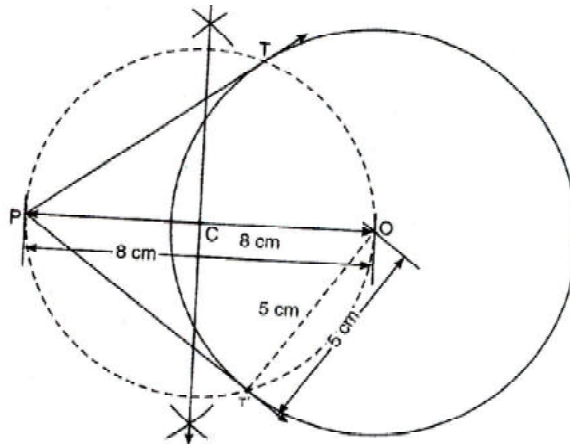
STEPS OF CONSTRUCTION :

Step-1 : Draw a circle with radius 5 cm whose centre is O.

Step-2 : Take a point P at a distance 8 cm from the centre O such that OP = 8cm.

Step-3 : Bisect the line segment OP at the point C such that OC =CP =4 cm.

Step-4 : Taking C as centre and OC as arc, draw a dotted circle to intersect the given circle at the points T and T'.

Step-5 : Join PT and PT'

PT and PT' are the required pair of tangents to the circle.

By measurement we obtain PT = PT' = 6.2 cm (Answer)

Verification: PT = PT' = $\sqrt{8^2 - 5^2}$ = $\sqrt{64 - 25}$ = $\sqrt{39}$ = 6.2 cm (Answer)

Ex.4 Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation. **[NCERT]**

Sol.
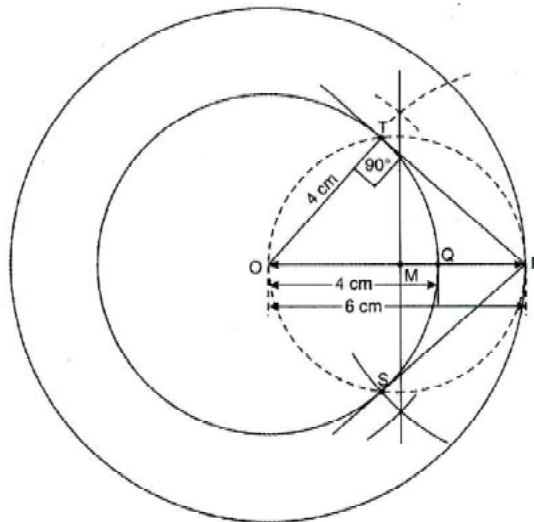
Steps of Construction:

Step 1 : Draw two concentric circles with centre O and radii 4 cm and 6 cm such that OP = 6 cm, OQ = 4 cm.

Step 2 : Join OP and bisect it at M. i.e. M is the mid-point of OP i.e. OM = PM = 3 cm.

Step 3 : Taking M as centre with OM as radius draw a circle intersecting the smaller circle in two points namely T and S.

Step 4 : Join PT and PS.

PT and PS are the required tangents from a point P to the smaller circle, whose radius is 4 cm. By measurement: PT = 4.5 cm.

Verification. OTP is right △ at T

$$OP^2 = OT^2 + PT^2$$
$$6^2 = 4^2 + PT^2 \quad \Rightarrow \quad PT^2 = 36 - 16 = 20$$

PT = $\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$ = 2 x 2.24 = 4.48 cm.

EXERCISE – I — UNSOLVED PROBLEMS

Q.1 Draw a line segment of length 7 cm and divide it in the ratio 3 : 2. Measure the two parts. Justify your construction.

Q.2 Draw a line segment of length 7.8 cm and divide it in the ratio 5 : 8. Measure the two parts.

Q.3 Construct a triangle with sides 5 cm, 6 cm and 7 cm and then construct another triangle similar to it whose sides are $\dfrac{2}{3}$ of the corresponding sides of the first triangle.

Q.4 Construct a triangle ABC whose sides are 5 cm, 12 cm and 13 cm. Construct another triangle similar to $\triangle$ABC and with sides $\dfrac{3}{5}$ the of the corresponding sides of the given triangle.

Q.5 Draw right triangle in which the sides (other than hypotenuse) are of lengths 3 cm and 4 cm. Then construct another similar triangle whose sides are $\dfrac{5}{3}$ times the corresponding sides of the given triangle.

Q.6 Construct a triangle with sides 5 cm, 6.5 cm and 7.6 cm and then construct another triangle similar to it whose sides are $\dfrac{7}{5}$ of the corresponding side of the first triangle.

Q.7 Construct a triangle similar to a given triangle with sides 6 cm, 7cm, and 8 cm, and whose sides are 1.4 times the corresponding sides of the given triangle.

Q.8 Draw a triangle with side BC = 6cm, $\angle$B = 45^0 and $\angle$A = 105^0. Then construct a similar triangle whose sides are $\dfrac{4}{3}$ times the corresponding sides of $\triangle$ABC.

Q.9 Construct a $\triangle$ABC in which AB = 5 cm, $\angle$B = 60^0 and altitude CD = 3 cm. Construct $\triangle$AQR similar. to $\triangle$ABC such that each side of $\triangle$AQR is 1.5 times that of the corresponding side of $\triangle$ABC.

Q.10 Draw a circle of radius 3cm. Take a point P on it. Construct a tangent to the circle at the point P. Also write the steps of construction.

Q.11 Draw a circle of radius 3.5 cm. Construct two tangents to it inclined at an angle of 60^0 to each other.

Q.12 Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60^0.

Q.13 Draw a circle of radius 3.5 cm. From a point P outside the circle at a distance of 6 cm from the centre of the circle, draw two tangents to the circle.

Q.14 Draw a circle of radius 3 cm. From a point 5 cm away from the centre of the circle. draw two tangents to the circle. Measure the lengths of the tangents.

Q.15 Draw a circle of diameter 12 cm. From a point 10 cm away from its centre, construct a pair of tangents to the circle. Measure the lengths of the tangent segments.

Q.16 Draw a circle of radius 3 cm. Take two points P and Q on one its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.

Q.17 Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation.

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Q.18 Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.

Q.19 Draw a circle with the help of a bangle. Take a point outside the circle Construct the pair of tangents from this point to the circle.

Q.20 Construct a triangle ABC similar to a given equilateral triangle PQR, with side 5 cm such that each of its side is $\frac{6}{7}$th of the corresponding side of ∆PQR. Also draw the circum circle of ∆ABC.

Q.21 Construct a triangle of sides 4 cm, 5 cm and 6 cm and then a triangle similar to it whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.

Q.22 Construct a triangle with sides 5 cm, 6 cm and 7 cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle.

Q.23 Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are $1\frac{1}{2}$ times the corresponding sides of the isosceles triangle.

Q.24 Draw a triangle ABC with side BC = 6 cm, AB = 5 cm and ∠ABC = 60°. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.

Q.25 Draw a triangle ABC with side BC = 7 cm, ∠B = 45°, ∠A = 105°. Then, construct a triangle whose sides are $\frac{4}{3}$ times the corresponding sides of ∆ABC.

Q.26 Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm. Then construct another triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the given triangle. In each of the following, give also the justification of the construction.

Q.27 Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle.

Q.28 Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation.

Q.29 Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.

Q.30 Let ABC be a right triangle in which AB = 6 cm, BC = 8 cm and ∠B = 90°. BD is the perpendicular from B on AC. The circle through B, C, D is drawn. Construct the tangents from A to this circle.

ANSWER KEY			
2.	3 cm, 4.8 cm	**14.**	4 cm (each)
15.	8 cm (each)	**17.**	4.5 cm

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EXERCISE – II · BOARD PROBLEMS

Q.1 Draw a line segment AB = 7 cm. Divide it internally in the ratio of (i) 3 : 5, (ii) 5 : 3. **[2000 C]**

Q.2 From a point P on the circle of radius 4 cm, draw a tangent to the circle with using the centre. Also write the steps of construction. **[2000]**

Q.3 Draw a circle of radius 4.5 cm. Take a point P on it. Construct a tangent at the point P without using the centre of the circle. Write the steps of construction. **[2001]**

Q.4 Divide a line segment of length 5.6 cm internally in the ratio (i) 3 : 2 (ii) 2 : 3. **[2001]**

Q.5 Construct a ∆ABC in which base AB = 6 cm, ∠C = 60° and the median CD = 5 cm. Construct a ∆AB'C' similar to ∆ABC with base AB' = 8 cm. **[2002]**

Q.6 Draw a circle of radius 3.5 cm. From a point P on the circle draw a tangent to the circle without using its centre. **[2003]**

Q.7 Draw a circle of radius 5 cm. Take a point P on it, without using the centre of the circle, construct a tangent at the point P. Write the steps of construction also. **[2003]**

Q.8 Draw a circle of diameter 12 cm. From a point P, 10 cm away from its centre, construct a pair of tangent to the circle. Measure the lengths of the tangent segments. **[2004 C]**

Q.9 Draw a circle of radius 3.5 cm. From a point P outside the circle at a distance of 6 cm from the centre of circle, draw two tangents to the circle. **[2005]**

Q.10 Construct a ∆ABC in which AB = 6.5 cm, ∠B = 60° and BC = 5.5 cm. Also construct a triangle AB'C' similar to ∆ABC, whose each side is $\frac{3}{2}$ times the corresponding side of the ∆ABC. **[Delhi-2008]**

Q.11 Draw a ∆ABC with side BC = 6 cm, AB = 5 cm and ∠ABC = 60°. Construct a ∆AB'C' similar to ∆ABC such that sides of ∆AB'C' are $\frac{3}{4}$ of the corresponding sides of ∆ABC. **[AI-2008]**

Q.12 Draw a right triangle in which the sides containing the right angle are 5 cm and 4 cm. Construct a similar triangle whose sides are $\frac{5}{3}$ times the sides of the above triangle. **[Foreign-2008]**

Q.13 Construct a ∆ABC in which BC = 6.5 cm, AB = 4.5 cm and ∠ABC = 60°. Construct a triangle similar to this triangle whose sides are $\frac{3}{4}$ of the corresponding sides of triangle ABC. **[Delhi-2008]**

Q.14 Draw a right triangle in which sides (other than hypotenuse) are of lengths 8 cm and 6 cm. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the first triangle. **[AI-2009]**

Q.15 Draw a circle of radius 3 cm. From a point P, 6 cm away from it's centre, construct a pair of tangents to the circle. Measure the lengths of the tangents. **[Foreign-2009]**

Q.16 Construct a triangle ABC in which AB = 8 cm, BC = 10 cm and AC = 6 cm. Then construct another triangle whose sides are $\frac{4}{5}$ of the corresponding sides of ∆ABC. **[AI-2010]**

Q.17 Construct a triangle ABC in which BC = 9 cm, ∠B = 60° and AB = 6 cm. Then construct another triangle whose sides are $\frac{2}{3}$ of the corresponding sides of ∆ABC. **[AI-2010]**

Q.18 Construct a triangle ABC in which BC = 8 cm, ∠B = 60° and ∠C = 45°. Then construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ∆ABC. **[AI-2010]**